Algorithmic Combinatorics
Exercises discussed on March 25, 2019

13. Let the Euler operator be defined as $\theta_x = xD_x$. Show that

$$\theta_x^{\underline{k}} f(x) = \theta_x(\theta_x - 1)\cdots(\theta_x - k + 1)f(x) = x^k f^{(k)}(x).$$

14. Prove Theorem 2.10 (Multiplicative Inverse):
Let $a(x) \in \mathbb{K}[\![x]\!]$. Then there exists a series $b(x) \in \mathbb{K}[\![x]\!]$ with $a(x)b(x) = 1$ if and only if $a(0) \neq 0$.

15. Let $(a_n(x))_{n\geq 0}, (b_n(x))_{n\geq 0}$ be convergent sequences of formal power series with respective limits $a(x), b(x) \in \mathbb{K}[\![x]\!]$.

    Show that then also $(c_n(x))_{n\geq 0}$ wiht $c_n(x) = a_n(x) + b_n(x)$ is a convergent sequence of formal power series with limit $a(x) + b(x)$.

16. Prove the reflection formula, i.e., show that for x an indeterminate and $k \in \mathbb{N}$:

$$\binom{x}{k} = (-1)^k \binom{k - x - 1}{k}.$$

17. Give a direct proof of the binomial theorem in the ring of formal power series, i.e., show that for $f(x) \in \mathbb{C}[\![x]\!]$ with $f(0) = 0$ and $\lambda \in \mathbb{C}$ that

$$(1 + f(x))^\lambda := \sum_{n\geq 0} \binom{\lambda}{n} f(x)^n \in \mathbb{C}[\![x]\!].$$