Algorithmic Combinatorics
Exercises discussed on March 11, 2019

1. Try to find a "pebble proof" for $\sum_{k=0}^{n}(2k+1) = (n+1)^2$.
2. Let $f : \mathbb{Z} \to \mathbb{C}$ and $a, b \in \mathbb{Z}$ with $a \leq b$.
   (a) Show that
   $$\sum_{k=a}^{b}\big(f(k+1) - f(k)\big) = f(b+1) - f(a).$$
   (b) Assume additionally that $f(k) \neq 0$ for all $a \leq k \leq b$. Show that
   $$\prod_{k=a}^{b}\frac{f(k+1)}{f(k)} = \frac{f(b+1)}{f(a)}.$$
3. Determine a closed form representation for the following sums:
   (a) $\sum_{k=1}^{n} k^2$
   (b) $\sum_{k=1}^{n} k^3$
4. Determine a closed form representation for the product
   $$p(n) = \prod_{k=2}^{n}\left(1 - \frac{1}{k^2}\right).$$
5. What is the worst-case choice of Pivot elements for Quicksort and what is the number of comparisons needed in that case?
6. Let
   $$g(n) = \sum_{k=0}^{n-1}\frac{2k}{(k+1)(k+2)}.$$
   Show that
   $$g(n) = 2H_n + \frac{4}{n+1} - 4.$$