

In[1]:= << RISC`GeneratingFunctions`

Package GeneratingFunctions version 0.8 written by Christian Mallinger  
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In[2]:= ? REPlus

RecurrenceEquationPlus[re1,re2,a[n]] gives a recurrence equation that is satisfied by the sum of solutions of the recurrences re1 and re2. All recurrences are given in a[n].

Alias: REPlus

See also: REInfo, DEPlus

(\* defining the sequences of Perrin and Lucas numbers \*)

In[3]:= perrin = {a[n+3] - a[n+1] - a[n] == 0, a[0] == 3, a[1] == 0, a[2] == 2};  
lucas = {a[n+2] - a[n+1] - a[n] == 0, a[0] == 2, a[1] == 1};

(\* computing a recurrence for the sum of Perrin and Lucas numbers \*)

In[5]:= REPlus[perrin, lucas, a[n]]

Out[5]= {a[n] + 2 a[1+n] - 2 a[3+n] - a[4+n] + a[5+n] == 0,  
a[0] == 5, a[1] == 1, a[2] == 5, a[3] == 7, a[4] == 9}

In[6]:= ? RE\*

▼ RISC`GeneratingFunctions`

RE	RE2L	REHadamard	REInterlace	REPlus	RESubsequence
RE2DE	RECauchy	REInfo	REOut	REShadow	

In[7]:= ? RESubsequence

RecurrenceEquationSubsequence[re,a[n],m\*n+k] gives a recurrence that is satisfied by a subsequence of the form a[m\*n+k] of every solution a[n] of the input recurrence re.

Alias: RESubsequence

See also: REInfo, REInterlace

(\* Computing a recurrence for all odd-indexed Lucas numbers \*)

```
In[8]:= RESubsequence[lucas, a[n], 2 n + 1]
```

```
Out[8]= {a[n] - 3 a[1 + n] + a[2 + n] == 0, a[0] == 1, a[1] == 4}
```

(\* Cauchy product of Perrin and Lucas numbers \*)

```
In[9]:= RECauchy[perrin, lucas, a[n]]
```

```
Out[9]= {a[n] + 2 a[1 + n] - 2 a[3 + n] - a[4 + n] + a[5 + n] == 0,
a[0] == 6, a[1] == 3, a[2] == 13, a[3] == 20, a[4] == 34}
```

(\* defining the Fibonacci sequence \*)

```
In[10]:= fib = {F[n + 2] - F[n + 1] - F[n] == 0, F[0] == 0, F[1] == 1};
```

(\* Computing the recurrence for the Cauchy product of Fibonacci numbers with the constant sequence 1, i.e., for the partial sum of Fibonacci numbers \*)

```
In[11]:= RECauchy[fib, {F[n] == 1}, F[n]]
```

```
Out[11]= {F[n] - 2 F[2 + n] + F[3 + n] == 0, F[0] == 0, F[1] == 1, F[2] == 2}
```

(\* Perrin and Lucas numbers are built-in \*)

```
In[12]:= ? Perrin
```

Perrin[n] gives the nth Perrin number.

```
In[13]:= ? LucasL
```

LucasL[n] gives the Lucas number  $L_n$ .

LucasL[n, x] gives the Lucas polynomial  $L_n(x)$ . >>

(\* creating data for guessing \*)

```
In[14]:= dataL = Table[LucasL[nn], {nn, 0, 20}];
```

```
dataP = Table[Perrin[nn], {nn, 0, 20}];
```

```
In[16]:= ? GuessRE
```

(\* Guessing the recurrence for Lucas numbers \*)

```
In[17]:= GuessRE[dataL, a[n]]
```

```
Out[17]= {{-a[n] - a[1 + n] + a[2 + n] == 0, a[0] == 2, a[1] == 1}, ogf}
```

(\* Guessing the recurrence for Perrin numbers → need to increase the maximal order used! \*)

```
In[18]:= GuessRE[dataP, a[n]]
```

```
Out[18]= FAIL
```

```
In[19]:= GuessRE[dataP, a[n], {0, 3}, {0, 0}]
```

```
Out[19]= {{-a[n] - a[1 + n] + a[3 + n] == 0, a[0] == 3, a[1] == 0, a[2] == 2}, ogf}
```