

## Exercises discussed on June 21, 2011

(BP13) Prove the following facts about Euler's totient function  $\varphi$ : For  $m, n \in \mathbb{N}$

(a)  $\gcd(m, n) = 1 \Rightarrow \varphi(mn) = \varphi(m)\varphi(n)$

(b)  $\varphi(n) = n \prod_{\substack{p|n \\ p \text{ prime}}} \left(1 - \frac{1}{p}\right)$

(c)  $\sum_{\substack{d=1 \\ d|n}}^n \varphi(d) = n$

(HW47) How many necklaces with exactly 3 red, 1 green and 2 black beads are there?

(HW48) Let  $G = (G, \circ)$  be a group and  $H \leq G$ . Show that then

$$H \times G \rightarrow G, \quad (h, g) \mapsto hg := hgh^{-1}$$

is a group action.

(HW49) (Refined Cauchy-Frobenius Lemma) Let  $X =_G X$ ,  $H \leq G$  and  $T_G$  be a set of representatives of conjugacy classes of  $G$  ( $\bigcup_{g \in T_G} C^G(g) = G$ ). Show that then

$$|H \backslash X| = \frac{1}{|H|} \sum_{g \in T_G} |C^G(g) \cap H| \cdot |X_g|.$$