Exercises discussed on May 31, 2011

(BP12) Let X be a finite set and $Y = \{y_1, \ldots, y_n\} \subseteq X$ and consider the group action

$$S_X \times X \to X, \quad (\pi, x) \mapsto \pi(x)$$

Let $\bigcup_{i \in I} X_i = X$ be the induced equivalence relation with respect to the group action $G_Y \times X \to X$. Show that then $|I| = 2$, say $I = \{1, 2\}$ where $X_1 = Y$ and $X_2 = X \backslash Y$.

(HW41) Prove that if G is finite and $H \subseteq G$ then $|G \backslash H| = \frac{|G|}{|H|}$.

(HW42) Consider the Polya action on simple labeled graphs, i.e.,

$$S_n \times \{0,1\}^{\binom{[n]}{2}} \to \{0,1\}^{\binom{[n]}{2}}, \quad (\pi, f) \mapsto \pi f = f \circ \overline{\pi}^{-1}$$

Use the Cauchy-Frobenius lemma to determine $|S_n \backslash \{0,1\}^{\binom{[n]}{2}}|$ for (a) $n = 3$ (b) $n = 4$.

(HW43) Consider the analogous problem for necklaces with 5 beads in at most 3 colors.

(HW44) Same as in the previous homework, but now counting bracelets. (Note: here D_5 instead of C_5 is the acting group.)