Exercises discussed on May 17, 2011

(BP10) Define group actions to specify the equivalence classes in cases 3A and 4A in the twelve fold way.

(HW36) Show that the induced permutation representation $\delta\colon G \to S_G$ is an injective group homomorphism.

(HW37) Prove the following proposition:
Let X be a G-set and $\delta : G \to S_X$ be the induced permutation representation. Then

(i) δ is a group homomorphism

(ii) $\overline{G} := \delta(G) \leq S_X$ acts on X as a subgroup of the permutation group.

(iii) $G\backslash\backslash X = \overline{G}\backslash\backslash X$

(BP11) With the same prerequisites as in (HW37) show that

(iv) $\mathrm{Ker}\delta = \{g \in G \mid \delta(g) = 1_{S_X}\}$ is a normal subroup of G

(v) $\overline{G} \cong G/_{\mathrm{Ker}\delta}$

(HW34) (d) Apply Frobenius lemma to the necklace problem.