Exercises discussed on May 3, 2011

(BP8) Prove for $n \geq 1$

$$p(n) - [p(n-1) + p(n-2)] + [p(n-5) + p(n-7)] + \cdots +$$
$$+ (-1)^m \left[p\left(n - \frac{m(3m-1)}{2}\right) + p\left(n - \frac{m(3m+1)}{2}\right)\right] + \cdots = 0,$$

implement it and compute $p(721)$ with it.

(HW27) Let $P_{dist}(n)$ denote the number of partitions into distinct parts and $P_{odd}(n)$ the number of partitions into odd parts. Give a combinatorial proof of $P_{dist}(n) = P_{odd}(n)$ for $n \in \mathbb{N}$.

(HW28) For a set $M \neq \emptyset$ define the map $S_M \times M \to M$, $(\pi, m) \to \pi m$, where $\pi m := \pi(m)$. Prove that this map is a group action.

(HW29) Prove that for any $\pi \in S_n$ $(n \in \mathbb{N}^*)$

$$\binom{[n]}{2} = \left\{ \{\pi(i), \pi(j)\} \mid \{i, j\} \in \binom{[n]}{2} \right\}.$$

(HW30) Prove that $\mathrm{GL}_{\mathbb{F}}(n) \times \mathbb{F}^n \to \mathbb{F}^n,\ (A, v) \mapsto Av$ is a group action.

(HW31) Find at least three other group actions.