Exercises discussed on April 12, 2011

(HW22) Show that the map

$$f : \mathrm{Comp}(n,k) \to \mathrm{Sets}(n-1,k-1),$$
$$(a_1, a_2, \dots, a_k) \mapsto \{a_1, a_1+a_2, \dots, a_1+a_2+\cdots+a_{k-1}\}$$

is a bijection.

(HW23) Derive a closed form for the number $C_1(n)$ of compositions on $n \geq 1$ in which no ones appear (e.g., $c_1(5):\ (5),(3,2),(2,3)$).

(BP7) Let $C_{even}(n)$ for $n \geq 2$ be the number of compositions of n with an even number of even parts. Find a closed form for $C_{even}(n)$.
(E.g., $C_{even}(4) = 4,\ (2,2),(3,1),(1,3),(1,1,1,1)$)

(HW24) Prove that $\sum_{k=1}^{x} p(n,k) = p(n+x,x)$ for $n, x \geq 1$.

(HW25) Give a combinatorial proof of $p(n,x) = p(n-1,x-1) + p(n-x,x)$ for $1 \leq x \leq n$.

(HW26) Let $\bar{p}(n,k)$ denote the number of partitions of n into maximally k parts and $p'(n,k)$ denote the number of n into parts of size $\leq k$. Show that for $k \geq 1$

$$\sum_{n \geq 0} \bar{p}(n,k) q^n = \sum_{n \geq 0} p'(n,k) q^n = \frac{1}{(1-q)(1-q^2)\cdots(1-q^k)}.$$