Exercises discussed on March 22, 2011

(BP2) Find a rank-function for Lists(n), $n \in \mathbb{N}$. I.e., find a bijection $r : \text{Lists}(n) \to [n!]$ $(= [|\text{Lists}(n)|])$ such that there is a nice rule to compute $r^{-1}(n)$.

(HW6) Find a rank-function for $[n]^{[k]}$ $(k, n \in \mathbb{N}^*)$. I.e., find a bijection $r : [n]^{[k]} \to [n^k]$ s.t. r^{-1} is easy to compute.

(BP3) Prove that the Prüfer map is a bijection.
Hint: To find the inverse of Prüfer's map look at concrete examples and at the edge degrees of the vertices.

(HW7) Unlabeled trees: How many unlabeled trees over 6 vertices are there?

(HW8) Unlabeled rooted trees: How many unlabeled rooted trees over 6 vertices are there?

(HW9) The winner of a tennis tournament is obtained by following a KO-system. If 128 players start, how many matches are played?

(HW10) Given a chocolate bar of $m \times n$ rectangular shape, which strategy do you apply to obtain the mn basic pieces by a minimal number of (straight line) partitions?

(HW11) Given a heap of $2k$ $(k \geq 1)$ coins. Player A splits it into 2 small heaps. Player B chooses one of the resulting heaps and splits it again into 2 smaller heaps. Then A chooses one of these heaps, etc.

The player who can carry out the last split into 2 non-heaps wins. Analyse the winning chances for A and B!

(HW12) Disect a planar n-gon $(n \geq 1)$ by introducing succesively non-intersecting diagonals. Does one always obtain $n - 2$ triangles?

(HW13) Given a complete binary (labeled/unlabeled) rooted tree with m leaves $(m \geq 2)$. Determine all possibilities for the number of inner nodes (non-leaves).