Exercises discussed on March 15, 2011

(HW1) Prove Lists(n,n) = Lists(n).

(HW2) Show that

$$\binom{x+1}{k} = \binom{x}{k} + \binom{x}{k-1}, \qquad k \in \mathbb{Z},\ x \in R.$$

(HW3) Give a combinatorial proof of

$$1 + 2 + \cdots + (n-1) = \binom{n}{2}.$$

(HW4) Give a combinatorial proof of

$$\sum_{k=0}^{n} \binom{x}{k}\binom{y}{n-k} = \binom{x+y}{n}, \qquad x, y, n \in \mathbb{N}.$$

(HW5) Give a combinatorial proof of

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k, \qquad n \in \mathbb{N}, \quad x \in \mathbb{C}.$$

(BP1) Let R be an integral domain, $n \in \mathbb{N}$, $S \subset R$ a finite subset with $|S| = s$ and $r \in R[x_1, \ldots, x_n]$ with total degree $d \in \mathbb{N}$. Then

(i) If $r \neq 0$, then r has at most ds^{n-1} zeroes in S^n.

(ii) If $s > d$ and r vanishes on S^n, then $r = 0$.

Prove this statement.