F. Winkler solve by 1 June 2010

Commutative Algebra & Algebraic Geometry
SS 2010

(27) Let $\mathcal{C} = V(f)$, where $f(x, y) = y^2 - x^3 - x^2$. Is the rational function $\varphi = y^3/(x + 1)$ on $\mathcal{C}$ regular at the point $(-1, 0)$? Is φ a regular function on $\mathcal{C}$?

(28) Given a rational function φ on a variety V. How can one decide algorithmically whether φ is regular on V?

(29) Prove: *A form in two variables $f(x, y)$ over an algebraically closed field splits completely into linear factors.*