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Commutative Algebra & Algebraic Geometry
SS 2010

(19) Determine a linear change of coordinates $L : \mathbb{C}^2 \to \mathbb{C}^2$ such that $L(f) = \tilde{f}$ is a Noether normalization of

$$f(x, y, z) = xz^3 - x^2y^2 - 2xyz$$

w.r.t. the variable x; i.e. $\tilde{f}$ should contain the term x^4.

(20) Consider the elliptic curve $\mathcal{E}$ defined by $e(x, y) = y^2 - x^3 + x$ (as in Example 1.3). Determine two different linear changes of coordinates L_1, L_2 of $\mathbb{A}^2(\mathbb{C})$, s.t. the point $P = (1, 1)$ is on $L_i(\mathcal{E})$, $i = 1, 2$. Prove that the L_i are indeed linear changes of coordinates, i.e. invertible linear maps.

(21) Primary ideals are not necessarily powers of prime ideals. In $\mathbb{Z}[x]$ consider the ideals

$$I = \langle 4, x \rangle, \quad J = \langle 2, x \rangle \ .$$

(a) Show that I is not prime; J is a prime divisor of I.
(Optional: J is the only proper non-trivial divisor of I.)
(b) I is not a power of J.

(22) Let R be a commutative ring with 1, and let I, J be ideals in R.
Show: *if I is prime and J is primary with $J \subseteq I$, then also $\sqrt{J} \subseteq I$* (Theorem 4.3.1(ii)).