F. Winkler solve by 20 April 2010

Commutative Algebra & Algebraic Geometry
SS 2010

(11) Exhibit a maximal chain of irreducible algebraic sets in $\mathbb{A}(\mathbb{C})^3$.
What is a corresponding maximal chain of properly ascending ideals in $\mathbb{C}[x, y, z]$?

(12) Prove that the half circle $C_{1/2} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \text{ and } y \geq 0\}$ is **not** an algebraic set in $\mathbb{R}^2$.

(13) Give an example of a commutative ring with 1, which is **not** Noetherian.

(14) Let R be a commutative ring with 1, and I an ideal in $R[x]$. Let J consist of all leading coefficients of elements in I plus 0. Prove that I is an ideal in R.
If I is a prime ideal, is then also J a prime ideal?