F. Winkler solve by 15 June 2010

Commutative Algebra & Algebraic Geometry
SS 2010

(33) Determine the intersection multiplicity of the curves defined by f_1 and f_2 in Example 7.1.3 in the lecture notes.

(34) Consider the linear system $\mathcal{S}_3$ of curves of degree 3.
  (a) Is it possible to determine, for any given 3 points P_1, P_2, P_3 in $\mathbb{P}^2(\mathbb{C})$, to find an element $\mathcal{C} \in \mathcal{S}_3$ having all these points as double points? If so, then determine such a curve for the points $(0:0:1), (0:1:1), (1:0:1)$.
  (b) Is it possible to determine, for any given 4 points P_1, P_2, P_3, P_4 in $\mathbb{P}^2(\mathbb{C})$, to find an element $\mathcal{C} \in \mathcal{S}_3$ having all these points as double points? If so, then determine such a curve for the points $(0:0:1), (0:1:1), (1:0:1), (1:1:1)$.

(35) Consider the linear system of quartic curves

$$\mathcal{S} = \{\mathcal{C} \text{ defined by } h \mid a, b, c, d, e \in \mathbb{C}\},$$

where

$$h(x, y, z) = ax^4 + bx^3y + cx^2yz + dxz^3 + ey^2z^2 - (a + b + c + d + e)z^4.$$

Which base points (with which muliplicities) does $\mathcal{S}$ have?

(36) Determine the genus (or, if there are non-ordinary singularities, give a genus bound) for the curves in Example 7.1.3 in the lecture notes.