State revival in spin networks, graphs and orthogonal polynomials

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Abstract: This talk will offer a survey of old and new results on the revival of quantum states along spin networks. I shall concentrate on two protocols: 1. Perfect State Transfer (PST) that has a qbit transported from one site to another with probability one and 2. Fractional Revival (FR) at two sites which sees an initial state being reproduced periodically at two locations and is tantamount to entanglement generation. How to design networks that enact these processes is the basic question.

I shall focus first on chains or weighted paths and explain how this engineering issue amounts to inverse spectral problems where orthogonal polynomials play a central role. I shall stress that end-to-end PST requires that the associated Jacobi matrix be persymmetric or mirror-symmetric. Examples that involve the Krawtchouk, para-Krawtchouk, dual Hahn univariate polynomials will be discussed.

Association schemes will be brought to bear on the analysis. Here the connection with orthogonal polynomials will arise through the Bose-Mesner algebra. I shall explore in certain instances the possibility that the dynamics on paths admit lifts to quantum walks on graphs of certain schemes.

I shall further present a model identified recently and based on the $q = -1$ limit of the dual q-Hahn polynomials, that exhibits asymmetric PST over even sites and FR between the odd ones. We shall use the spectral theory of orthogonal polynomials to identify how mirror symmetry is replaced in this case.

With an eye to looking at PST and FR in two-dimensional spin networks, we shall discuss the unfamiliar ordered 2-Hamming scheme and uncover its connection to the bivariate Krawtchouk polynomials of Tratnik type. We shall finally indicate how a 2-dimensional spin network with interesting PST and FR properties can be identified from projecting a quantum walk on a weighted graph of this scheme onto the plane.

The talk will end with indications of additional mathematical and physical connections.

Based on work with: P.-A. Bernard (Montreal), M. Christandl (Copenhagen), G. Coutinho (Belo Horizonte), V. X. Genest (Boston), E. Loranger (Montreal), H. Miki (Tokyo), C. Tamon (Postdam, US), S. Tsujimoto (Kyoto), A. Zhedanov (Beijing), H. Zhan (Montreal)