Recurrence equations and their classical orthogonal polynomial solutions on a quadratic or q-quadratic lattice

01.19 **Daniel Duviol Tcheutia**
(*Institute of Mathematics, University of Kassel, Germany*)
Time: Friday 26.07., 11:00 - 11:30, Room AM

Abstract: If $(p_n(x))_{n\geq 0}$ is an orthogonal polynomial system, then $p_n(x)$ satisfies a three-term recurrence relation of type

$$p_{n+1}(x) = (A_n x + B_n)p_n(x) - C_n p_{n-1}(x) \quad (n = 0, 1, 2, \ldots, p_{-1} \equiv 0),$$

with $C_n A_n A_{n-1} > 0$. On the other hand, Favard's theorem states that the converse is true. A general method to derive the coefficients A_n, B_n, C_n in terms of the polynomial coefficients of the divided-difference equations satisfied by orthogonal polynomials on a quadratic or q-quadratic lattice is recalled. If a three-term recurrence relation is given as input, the Maple implementations `rec2ortho` of Koorwinder and Swarttouw (1996) or `retode` of Koepf and Schmersau (2002) can identify its solution which is a (linear transformation of a) classical orthogonal polynomial system of a continuous, a discrete or a q-discrete variable, if applicable. The two implementations `rec2ortho` and `retode` do not handle classical orthogonal polynomials on a quadratic or q-quadratic lattice. Motivated by an open problem, submitted by Alhaidari during the 14th International Symposium on Orthogonal Polynomials, Special Functions and Applications, which will serve as application, the Maple implementation `retode` of Koepf and Schmersau is extended to cover classical orthogonal polynomial solutions on quadratic or q-quadratic lattices of three-term recurrence relations.