Three tales from one pocket

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Abstract: I plan to present (and connect) three at first glance unrelated results found in joint works with A. Borichev, A. Kononova, A Nishry and B. Weiss (arXiv:1409.2736, 1701.03407, 1902.00874, 1902.00872)

(1) As all of us know, the exponential decay of the series $\sum_{n\geq 0}(-1)^n x^n/n!$ is a result of incredible cancellations. Quite surprisingly, one can achieve a similar cancellation replacing the sequence $(-1)^n$ by a random complex-valued stationary correlated sequence provided that the origin does not belong to the closed convex hull of the support of the spectral measure of the sequence.

(2) Finitely valued random stationary sequences possess a striking spectral property: if the spectral measure of the sequence has a gap in its support then the sequence must be periodic.

(3) Given a non-negative measure ρ on the unit circle $\mathbb{T}$, the Szegő minimum problem is to find the quantity

$$e_n(\rho)^2 = \min_{q_1,\dots,q_n} \int_{\mathbb{T}} \left|1 + q_1 t + \ldots + q_n t^n\right|^2 \mathrm{d}\rho(t).$$

A celebrated result, first, proven by Szegő for absolutely continuous measures ρ, and then, independently, by Verblunsky and Kolmogorov in the general case, states that

$$\lim_{n\to\infty} e_n(\rho) = \exp\Big(\frac{1}{2}\int_{\mathbb{T}} \log(\mathrm{d}\rho/\mathrm{d}m)\,\mathrm{d}m\Big),$$

where m is the Lebesgue measure on $\mathbb{T}$ normalized by condition $m(\mathbb{T}) = 1$ and $\mathrm{d}\rho/\mathrm{d}m$ is the Radon-Nikodym derivative. Thus, $\lim_{n\to\infty} e_n(\rho) = 0$ if and only if the measure ρ has a divergent logarithmic integral. In spite of the classical nature of this result, little is known how properties of a measure ρ with divergent logarithmic integral affect the rate of decay of the sequence $e_n(\rho)$. I will present several quantitative results in that direction.