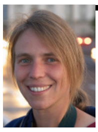
Orthogonal Polynomials, Special Functions, and Algorithms



Veronika Pillwein
(*Johannes Kepler Universität, Linz, Austria*)
Time: Tuesday 23.07., 14:00, Room AM

Abstract: Orthogonal polynomials and special functions are used in many different scientific fields and so are methods from symbolic computation. The study of special functions is part of classical mathematical theory. For instance, people have been deriving linear differential equations or difference equations satisfied by special functions or, starting from a differential equation, determining recurrence relations of coefficient sequences since centuries. Many of the techniques used here are already algorithmic and have been implemented and extended in the recent decades.

A step beyond these classical methods has been initiated by Doron Zeilberger and his holonomic systems approach in the early 1990s. For instance Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ are a feasible input for these algorithms and linear relations between shifts in the parameters n, α, β and derivatives w.r.t. x can be found and proven automatically.

Some of these algorithms have been generalized to cover also certain non-holonomic input such as Stirling-type sequences. This approach is still closely related to Zeilberger's paradigm. Relying on classical symbolic algorithms such as Gröbner bases computations or Cylindrical Algebraic Decomposition, methods have been developed for finding and proving non-linear algebraic relations and for proving inequalities on special functions satisfying general types of recurrences.

Recently, we have extended algorithms for executing closure properties from D-finite functions (satisfying linear recurrence relations with polynomial coefficients) to DD-finite functions (satisfying linear differential equations with D-finite function coefficients). This is joint work with Antonio Jiménez-Pastor.

In this talk, we want to present different applications where some of these methods have been successfully applied without going into detail about the underlying algorithmic aspects.