Inverse Zeilberger's Problem

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Abstract: Given a proper hypergeometric term $F(n,k)$, Zeilberger's *Creative Telescoping algorithm* finds a linear recurrence with rational coefficients satisfied by the sequence $s_n = \sum_{k=0}^{n} F(n,k)$. In the context of solving recurrence equations, we consider here what might be called the *inverse Zeilberger's problem*: given a homogeneous linear recurrence with polynomial coefficients, find its solutions representable as definite sums of a certain form.

As a first step in this direction, we provide an algorithm which, given a linear recurrence operator L with polynomial coefficients, and a product of binomial coefficients of the form

$$F(n,k) \ = \ \prod_{i=1}^{m} \binom{a_i n + b_i}{k}$$

where a_i are positive integers and b_i are arbitrary constants, returns a linear recurrence operator L' with rational coefficients such that for any sequence y of the form $y_n = \sum_{k=0}^{\infty} F(n,k) h_k$, we have $Ly = 0$ if and only if $L'h = 0$. This enables us to find all such solutions y where h belongs to a class of holonomic sequences with a known algorithm for converting from recursive to explicit representation.