## MS12: General session for contributed talks

### Integrals of Mellin–Barnes type

**12.01** **Gopal Srinivasan**
(*Department of Mathematics, Indian Institute of Technology Bombay, India*)
**Time:** Monday 22.07., 10:30 - 11:00, Room HS 6

**Abstract:** Integrals involving ratios of products of gamma functions along vertical lines in the right half plane have been important since their appearance in 1890s featuring prominently in analytic number theory (in the works of Cahen, Pincherle and others) and also formed a point of departure for Barnes for his development of the theory of hypergeometric functions. The Meier G-functions are also representable as integrals of Mellin-Barnes type. These integrals feature in the work of Hecke in connection with certain problems in algebraic number theory. In this talk we shall look at these integrals from the point of view of pull backs of distributions in the sense of Laurant Schwartz via submersive maps deriving many classical well-known identities as corollaries.

### Discrete variations on an old special functions theme

**12.02** **Ciprian-Sorin Acatrinei**
(*NIPNE, Bucharest, Romania*)
**Time:** Monday 22.07., 11:00 - 11:30, Room HS 6

**Abstract:** A discretization scheme is discussed for field theories defined over $2+1$ and $3+1$ dimensional spaces, with particular emphasis on the special-functions-type solutions of their equations of motion. The space-time coordinates are assumed to be operators forming a non-commutative algebra; in the representation chosen, the radial coordinate becomes discrete, whereas the dependence on the other coordinates transfers to nonlocal field correlations. The discrete equations of motion and their complete solutions are presented in detail, together with their continuum/commutative limit. In $2+1$ dimensions one obtains a discrete extension of the Bessel and Neumann functions, passing through less known aspects of the Laguerre polynomials recurrence relation. In $3+1$ dimensions the relevant recurrence relation is the one satisfied by the Hahn polynomials.

### Symbolic evaluation of $hp$-FEM element matrices on simplices

**12.03** **Tim Haubold**
(*University of Hannover, Germany*)
**Time:** Monday 22.07., 11:30 - 12:00, Room HS 6

**Abstract:** In this talk we consider high-order finite element discretizations of linear elliptic boundary value problems. Following e.g. [1,2] a set of hierarchic basis functions on simplices is chosen. For an affine simplicial triangulation this leads to a sparse stiffness matrix. Moreover the $L_2$-inner product of the interior basis functions is sparse with respect to the polynomial order $p$. The construction relies on a tensor-product based construction with properly weighted Jacobi polynomials.

In this talk we present algorithms which compute the remaining non zero entries of mass- and stiffness matrix in optimal arithmetical complexity. In order to obtain this result, recursion fomulas based on symbolic methods [3] are used. The presented techniques can be applied not only to scalar elliptic problems in $H^1$ but also for vector valued problems in $H(\text{div})$ and $H(\text{curl})$, where an explicit splitting of the higher-order basis functions into solenoidal and non-solenoidal ones is used.

[1] Beuchler, Pillwein, Schöberl, Zaglmayr: *Sparsity Optimized High Order Finite Element Functions on Simplices*, 2012.
[2] Karniadakis, Sherwin: *Spectral/HP Element Methods for CFD*, 1999.
[3] Kauers: *SumCracker – A Package for Manipulating Symbolic Sums and Related Objects*, 2006.

## Orthogonal polynomials arising from the expansion of first degree polynomials

**12.04** **Shara Lalo**
(*Derby, UK*)
**Time:** Monday 22.07., 12:00 - 12:30, Room HS 6

**Abstract:** Expansions of univariate polynomials of the first degree with real and complex coefficients; naturally produce many well-known polynomials such as; Chebyshev, Fibonacci, Pell, Jacobsthal, Fermat polynomials and many new orthogonal polynomials. We will present new differential equations related to orthogonal polynomials arising from the expansion of first-degree polynomials. The roots, weights, generating functions and equivalent Rodriguez type formulas for the orthogonal polynomials will also be presented. This is joint work with Zagros Lalo.

## Hook length property of $d$-complete posets via $q$-integrals

**12.05** **Meesue Yoo**
(*Dankook University, South Korea*)
**Time:** Tuesday 23.07., 17:00 - 17:30, Room HS 5

**Abstract:** In this work, we prove the hook length property of the $d$-complete posets using the $q$-integral technique developed by Kim and Stanton. For a non-negative integer $n$, the generating function for the number of partitions of $n$ with no more than $k$ parts, $p_k(n)$, is given by

$$\sum_{n=0}^{\infty} p_k(n)q^n = \prod_{i=1}^{k} \frac{1}{1-q^i}.$$

Considering a partition of $n$ with no more than $k$ parts as an order-reversing map from a $k$-element chain to the set of non-negative integers such that the sum of images equals to $n$, Stanley extended this concept of partition and defined $P$*-partitions* of $n$. Then Stanley proved that the $P$-partition generating function for shapes has the hook length property. Proctor and Peterson figured out that the $d$-complete posets satisfy the hook length property, and Proctor showed that any connected $d$-complete poset $P$ can be uniquely decomposed into a slant sum of one element posets and irreducible components. Furthermore, he classified 15 disjoint classes of irreducible components and showed that these 15 disjoint classes exhaust the set of all irreducible components. We show that the $P$-partition generating function for each irreducible $d$-complete poset can be written as a $q$-integral and prove the hook length property of them by computing the $q$-integrals explicitly. This is a joint work with Jang Soo Kim.

## Recurrence equations involving different orthogonal polynomial sequences

**12.06** **Aletta Jooste**
(*University of Pretoria, South Africa*)
**Time:** Wednesday 24.07., 10:30 - 11:00, Room SH 02

**Abstract:** Every sequence of real polynomials $\{p_n\}_{n=0}^{\infty}$, orthogonal with respect to a positive weight function $w(x)$ on the interval $(a, b)$, satisfies a three-term recurrence equation. We discuss the role played by the polynomials *associated* to $p_n$, especially as coefficient polynomials in the three-term recurrence equation involving polynomials $p_n, p_{n-1}$ and $p_{n-m}, m \in \{2, 3, \ldots, n-1\}$. Furthermore, we show how Christoffel's formula is used to obtain mixed three-term recurrence equations involving the polynomials $p_n$, $p_{n-1}$ and $g_{n-m,k}, m \in \{2, 3, \ldots, n-1\}$, where the sequence $\{g_{n,k}\}_{n=0}^{\infty}, k \in \mathbb{N}_0$, is orthogonal with respect to $c_k(x)w(x) > 0$ on $(a, b)$ and $c_k$ is a polynomial of degree $k$ in $x$. The equations obtained can be used to study the location of the zeros of the appropriate polynomials.

## Completely monotonic Fredholm determinants

**12.07** **Ruiming Zhang**
(*College of Science, Northwest A&F University, Yangling, Shaanxi, China*)
**Time:** Wednesday 24.07., 11:00 - 11:30, Room SH 02

**Abstract:** This talk is based on a joint work with Professor Mourad Ismail. In this talk we discuss some monotonicity questions related to Fredholm matrices and operators. A function $f(x)$ is called completely monotonic if $(-1)^m f^{(m)}(x) > 0$. It is known that the expectation of having $m$ eigenvalues of a random Hermitian matrix in an interval is a multiple of $(-1)^m$ times the $m$-th derivative of a Fredholm determinant at $\lambda = 1$. In this work we extend the positivity to half-real line $(-\infty, 1]$, and we also study the completely monotonicity of some special functions which arise as Fredholm determinants.

## A generalization of the Silvester-Kac matrix

**12.08** **Mikhail Tyaglov**
(*Shanghai Jiao Tong University, China*)
**Time:** Wednesday 24.07., 11:30 - 12:00, Room SH 02

**Abstract:** Recently [2], C. da Fonseca et al considered a model for deposition and evaporation on discrete cells of a finite array of any dimension that led to a matrix equation involving a Sylvester-Kac type matrix. They found the eigenvalues and eigenvectors of that matrix and generalized some results of R. Askey [1] and O. Holtz [3]. In this talk, we discuss a somewhat novel approach that allows to generalize the results of all previous authors. More exactly, we find the eigenvalues and eigenvectors of the following $N \times N$ matrix:

$$\begin{pmatrix} 0 & \alpha & 0 & \dots & 0 & 0 & 0 \\ -N\gamma & \beta & 2\alpha & \dots & 0 & 0 & 0 \\ 0 & -(N-1)\gamma & 2\beta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (N-2)\beta & (N-1)\alpha & 0 \\ 0 & 0 & 0 & \dots & -2\gamma & (N-1)\beta & N\alpha \\ 0 & 0 & 0 & \dots & 0 & -\gamma & N\beta \end{pmatrix},$$

where $\alpha$, $\beta$, and $\gamma$ are arbitrary complex numbers, and $\gamma \neq 0$.

[1] R. Askey, Evaluation of Sylvester type determinants using orthogonal polynomials, in: H.G.W. Begehr, et al. (Eds.), Advances in Analysis, World Scientific, Hackensack, NJ, 2005, pp. 1–16.
[2] C. da Fonseca, D. Mazilu, I. Mazilu, T. Williams, The eigenpairs of a Sylvester-Kac type matrix associated with a simple model for one-dimensional deposition and evaporation, Appl. Math. Lett., 26, no. 12, pp. 1206–1211.
[3] O. Holtz, Evaluation of Sylvester type determinants using block-triangularization, in: H.G.W. Begehr, et al. (Eds.), Advances in Analysis, World Scientific, Hackensack, NJ, 2005, pp. 395–405.

## The fractional Green's function by Babenko's approach

**12.09** **Chenkuan Li**
(*Brandon University, Brandon, Manitoba, Canada*)
**Time:** Wednesday 24.07., 12:00 - 12:30, Room SH 02

**Abstract:** The goal of the current work is to derive the fractional Green's function for the first time in the distributional space $\mathcal{D}'(R^+)$ for the following fractional-order differential equation with constant coefficients

$$a_n u^{(\beta_n)}(x) + a_{n-1} u^{(\beta_{n-1})}(x) + \cdots + a_1 u^{(\beta_1)}(x) + a_0 u^{(\beta_0)}(x) = g(x).$$

Our new technique is based on Babenko's approach, without using any integral transforms, such as the Laplace transform, and the Mittag-Leffler functions. The results obtained are not only simpler but also more generalized than classical ones as they deal with distributions in Schwartz's sense. Furthermore, we provide several interesting applications of solving the fractional differential and integral equations by showing the convergence of double series based on gamma functions.

## New degenerate Eulerian polynomials

**12.10** **Orli Herscovici**
(*Department of Mathematics, Technion – Israel Institute of Technology, Haifa, Israel*)
**Time:** Friday 26.07., 10:30 - 11:00, Room HS 4

**Abstract:** We introduce a new generalization of the Eulerian polynomials based on the degenerate exponential function defined by Tsallis. Classical Eulerian polynomials can be defined by a few different generating functions. We present the generalizations of those generating functions, study properties of the degenerate Eulerian polynomials and present generalizations of some familiar identities which we have established in our preliminary work. Moreover, we give an explicit form of the coefficients of the degenerate Eulerian polynomials.

## The Gamma function and its inverse

**12.11** **David Jeffrey**
(*University of Western Ontario, London, Ontario, Canada*)
**Time:** Friday 26.07., 11:00 - 11:30, Room HS 4

**Abstract:** Some new results on expansions of the Gamma function will be presented. The functional inverse of the Gamma function has been little studied, although there are applications. The definition of the inverse will be discussed and its computation described.

## Discretization of generalized Chebyshev polynomials of (anti)symmetric multivariate sine functions

**12.12** **Adam Brus**
(*Czech Technical University, Prague, Czech Republic*)
**Time:** Friday 26.07., 11:30 - 12:00, Room HS 4

**Abstract:** The multivariate antisymmetric and symmetric trigonometric functions allow to generalize the classical Chebyshev polynomials to multivariate settings. The four classes of the multivariate polynomials, related to the symmetrized sine functions, are studied. For each of these polynomials, the weighted continuous and discrete orthogonality relations are shown. The related cubature formulas for numerical integration together with further model examples and properties of selected special cases are discussed.