MS04: Multivariate special functions related to Lie algebras

Organizer: Michael Schlosser (*Universität Wien, Vienna, Austria*)

In this mini-symposium recent developments on multivariate special functions related to Lie algebras, or root systems, will be considered. The topics include but are not restricted to symmetric functions (such as Macdonald polynomials, Macdonald-Koornwinder polynomials, etc.), integrable systems, related physical and combinatorial models, connections to representation theory, conformal field theory and character identities.

Intertwining operator for the dihedral group

04.01 **Yuan Xu**
(*University of Oregon, USA*)
Time: Thursday 25.07., 10:30 - 11:00, Room HS 6

Abstract: Dunkl operators associated to a dihedral group are a pair of differential-difference operators that generate a commutative algebra. The intertwining operator intertwines between this algebra and the algebra of ordinary differential operators. We will discuss an integral representation of the intertwining operator on a class of functions. As an application, closed formulas of the Poisson kernels are derived for sieved Gegenbauer polynomials and several related families of orthogonal polynomials.

The higher rank q-Bannai–Ito algebra and multivariate $(-q)$-Racah polynomials

04.02 **Hadewijch De Clercq**
(*Ghent University, Belgium*)
Time: Thursday 25.07., 11:00 - 11:30, Room HS 6

Abstract: The q-Racah polynomials are well-known to be bispectral, i.e. they can be defined through both a second-order q-difference equation and a three-term recursion relation. This bispectrality is described algebraically by the Askey–Wilson or Zhedanov algebra, and its counterpart under a transformation $q \to -q$, the so-called q-Bannai–Ito algebra. In this talk, I will explain how these connections can be generalized to multiple variables. We will construct a higher rank extension for the q-Bannai–Ito algebra by exploiting the Hopf algebraic structure of quantum groups. Then we will show how this novel algebra encodes the bispectrality of Gasper & Rahman's multivariate $(-q)$-Racah polynomials. More precisely, we will study how this algebra acts on the discrete series representation of the corresponding quantum group, and identify a class of canonical bases. Several such bases are in duality, in the sense that their overlap coefficients can be expressed as multivariate $(-q)$-Racah polynomials. Iliev's bispectral q-shift operators give rise to a discrete realization of the higher rank q-Bannai-Ito algebra. Finally, I will discuss the limit $q \to 1$, which suggests a construction for multivariate Bannai–Ito polynomials.

This is joint work with Hendrik De Bie.

A quantum algebra approach to multivariate Askey–Wilson polynomials

04.03 **Wolter Groenevelt**
(*Delft University of Technology, Netherlands*)
Time: Thursday 25.07., 11:30 - 12:00, Room HS 6

Abstract: In this talk we show that the multivariate Askey–Wilson polynomials introduced by Gasper and Rahman occur as matrix elements of representations of the quantum algebra $\mathcal{U}_q(su(1,1))$. From this interpretation several properties of the polynomials, e.g. orthogonality, can be obtained.

Elliptic extension of Gustafson's q-integral of type G_2

04.04 **Masahiko Ito**
(*University of the Ryukyus, Nishihara, Okinawa Prefecture, Japan*)
Time: Thursday 25.07., 12:00 - 12:30, Room HS 6

Abstract: I will present an evaluation formula for an elliptic beta integral of type G_2. The integral is expressed by a product of Ruijsenaars' elliptic gamma functions, and the formula includes that of Gustafson's q-beta integral of type G_2 as a special limiting case as $p \to 0$. The elliptic beta integral of type BC_1 by van Diejen and Spiridonov is effectively used in the proof of the evaluation formula. This is a joint work with M. Noumi.

Convolution identities arising from the Lie superalgebra $\mathfrak{osp}(1|2)$

04.05 **Erik Koelink**
(*Radboud University, Nijmegen, Netherlands*)
Time: Thursday 25.07., 15:30 - 16:00, Room HS 6

Abstract: There is a link between various sets of orthogonal polynomials and the representation theory of the Lie superalgebra $\mathfrak{osp}(1|2)$. We consider the irreducible unitary representations of $\mathfrak{osp}(1|2)$ which are the analogues of discrete series representations, for which explicit tensor product decompositions exist. By diagonalising an explicit operator of $\mathfrak{osp}(1|2)$ in the representations and in the two- and three-fold tensor products we get eigenfunctions of this operator in different ways; the coupled and the uncoupled way. The relations between these eigenvectors leads to the convolution identities, which can also be viewed as an identity for orthogonal polynomials in two variables. The polynomials involved are Bannai–Ito polynomials and super extensions of the Jacobi, Hahn and generalised Hermite polynomials. Using explicit realisations of the representation, we find a bilinear generating function involving Bessel functions.

This is based on joint work with Jean-Michel Lemay and Luc Vinet, both at CRM, U. de Montréal, Canada.

A nonsymmetric version of Okounkov's BC-type interpolation Macdonald polynomials

04.06 **Tom Koornwinder**
(*University of Amsterdam, Netherlands*)
Time: Thursday 25.07., 16:00 - 16:30, Room HS 6

Abstract: In 1998 Okounkov introduced BC-type interpolation Macdonald polynomials. These are symmetric Laurent polynomials which are determined, up to a constant factor, by their vanishing on interpolation points which depend on q and two additional parameters s and t. He also showed that Macdonald-Koornwinder polynomials can be explicitly expanded in terms of products of two such interpolation polynomials, one in the variable and one in the dual variable. This so-called binomial formula specializes in the one-variable case to the usual q-hypergeometric expression for Askey-Wilson polynomials. Furthermore, Okounkov's polynomials allow extra-vanishing, i.e., they vanish not just on the interpolation points, but also on an additional explicit point set.

The talk presents recent work joint with Disveld and Stokman (see arXiv:1808.01221) where we introduce a nonsymmetric version of Okounkov's polynomials. These are Laurent polynomials (no longer symmetric) characterized by their vanishing on interpolation points. The symmetric Okounkov polynomials can be expressed as a sum over the Weyl group for BC_n of the nonsymmetric polynomials. The existence proof of the nonsymmetric polynomials is by a nested induction process. There are experimental indications for extra-vanishing of the nonsymmetric polynomials.

Riesz distributions and the Wallach set in Dunkl theory

04.07 **Margit Rösler**
(*Paderborn University, Germany*)
Time: Thursday 25.07., 16:30 - 17:00, Room HS 6

Abstract: We introduce Riesz distributions associated with rational Dunkl operators of type A, which are closely related to the well-known Riesz distributions on symmetric cones, such as cones of positive-definite matrices. The study of these distributions relies on the rigorous foundation of a suitable Laplace transform in the Dunkl setting, which goes back to Macdonald but had been established so far only on a formal level. In particular, we shall present an analogue of a famous result of Gindikin for symmetric cones, which states that a Riesz distribution is actually a positive measure if and only if its index belongs to the so-called Wallach set. Besides the Laplace transform, Jack polynomial expansions play an important role in the proofs.

Multidimensional matrix inversions and elliptic hypergeometric series on root systems

04.08 **Hjalmar Rosengren**
(*Chalmers University of Technology and University of Gothenburg, Sweden*)
Time: Thursday 25.07., 17:00 - 17:30, Room HS 6

Abstract: Explicit matrix inversions give a powerful tool for studying hypergeometric series in one or several variables. We will discuss some new multidimensional matrix inversions and their applications to elliptic hypergeometric series on root systems. The talk is based on joint work with Michael Schlosser.

Multivariate Chebyshev polynomials in algebraic signal processing

04.09 **Bastian Seifert**
(*Universität Würzburg, Germany*)
Time: Friday 26.07., 10:30 - 11:00, Room HS 6

Abstract: Algebraic signal processing theory is a unified setting for various linear signal processing concepts. In this setting one can derive fast algorithms for the computation of Fourier transforms of suitable signal models based on a decomposition property of polynomials. In this talk we first give a short introduction to algebraic signal processing theory. Then we will explain why the multivariate Chebyshev polynomials associated to root systems are powerful building blocks for signal models. Furthermore we present a geometric interpretation of the fast algorithms corresponding to the multivariate Chebyshev polynomials.

Multivariate Meixner, Charlier and Krawtchouk polynomials

04.10 **Genki Shibukawa**
(*Kobe University, Japan*)
Time: Friday 26.07., 11:00 - 11:30, Room HS 6

Abstract: In a previous paper (Journal of Lie Theory 26 (2016) 439–477), we introduced a multivariate analogue of Meixner, Charlier and Krawtchouk polynomials and established their main properties; generating functions, orthogonality, difference equations (recurrence formulas). Our multivariate Meixner, Charlier and Krawtchouk polynomials are also regarded as 2 (or 1) parameter deformations of rational analogue for Macdonald polynomials of type A. However, our proofs are based on harmonic analysis on symmetric cones (special functions for matrix arguments) and all our results need a restriction condition for the coupling constant. Recently, we give new their proofs without using harmonic analysis on symmetric cones, and succeed in extending all our previous results for any coupling constant. We would like to talk about these recent advances.

Racah problems for the oscillator algebra and $\mathfrak{sl}_n$

04.11 **Wouter van de Vijver**
(*Ghent University, Belgium*)
Time: Friday 26.07., 11:30 - 12:00, Room HS 6

Abstract: We consider the tensor product of n copies of the oscillator algebra $\mathfrak{h}$. Using the Hopf structure and Casimir operator of $\mathfrak{h}$, we construct a subalgebra $\mathcal{R}_n(\mathfrak{h})$ in the same way the higher rank Racah algebra was constructed for $\mathfrak{su}(1,1)$ in [1]. One can embed the algebra $\mathcal{R}_n(\mathfrak{h})$ into $\mathfrak{sl}_{n-1}$ after an affine transformation of the generators by central elements. We study the connection between recoupling coefficients for $\mathfrak{h}$ and $\mathfrak{sl}_n$-representations. These coefficients turn out to be multivariate Krawtchouck polynomials. The relation with the Wigner-$3nj$ symbols for $\mathfrak{h}$ is explained. Flipping two factors in the tensor product is a symmetry of $\mathcal{R}_n(\mathfrak{h})$. This leads to an automorphism of $\mathfrak{sl}_{n-1}$. The corresponding group elements of $\mathrm{SL}(n-1)$ are constructed.

This is joint work with Nicolas Crampé and Luc Vinet.

[1] H. De Bie, V.X. Genest, L. Vinet, W. van de Vijver, A higher rank Racah algebra and the $(\mathbb{Z}_2)^n$ Laplace-Dunkl operator. *J. Phys. A: Math. Theor.* 51 025203 (20pp), 2018.