Hypergeometric transformations based on Hahn and Racah polynomials

02.10 **Robert S. Maier**
(*University of Colorado, Boulder, USA*)
Time: Tuesday 23.07., 11:00 - 11:30, Room AM

Abstract: Much as the Gauss hypergeometric function ${}_2F_1$ satisfies many transformation identities, the function ${}_3F_2$ can be quadratically and cubically transformed. For example, a ${}_3F_2$ with a parametric excess equal to $\frac{1}{2}$ or $-\frac{1}{2}$ may be quadratically transformed to a well-poised ${}_3F_2$ or a very well-poised ${}_4F_3$. Summation identities can be derived from such transformations by the classical technique of equating coefficients, or by Gessel–Stanton pairing. We show that the classical quadratic and cubic transformations of ${}_3F_2$ can extended: in the quadratic case, the parametric excess may be greater than $\frac{1}{2}$ or less than $-\frac{1}{2}$ by any natural number. The transformed functions now become hypergeometric functions of higher order, the added parameters of which make contact with the theory of orthogonal polynomials of a discrete argument. For instance, the added parameters can be the (negated) roots of certain dual Hahn or Racah polynomials, which are defined on a quadratic lattice; or in the cubic case, new polynomials with no evident orthogonal interpretation. Extended versions of summation identities of Whipple and Bailey can be derived from the extended transformations of ${}_3F_2$: for instance, extensions of Dougall's theorem on the sum of a 2-balanced, very well-poised ${}_7F_6(1)$.