Newton diagram for the positivity of ${}_1F_2$ hypergeometric functions and Askey-Szegő problem

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Abstract: Concerning the positivity inequality

$$\text{(P)} \qquad {}_1F_2\left[\begin{array}{c} a \\ b,c \end{array}\middle| -\frac{x^2}{4}\right] \geq 0 \qquad (x>0),$$

with $a>0$ fixed, we prove that if the parameter pair (b,c) belongs to certain hyperbolic region in $\mathbb{R}^2_+$ containing the Newton diagram associated to $\{(a+1/2,2a),(2a,a+1/2)\}$, then (P) holds true. As an application, we consider the Askey-Szegő problem, related with

$$\int_0^x t^{-\beta} J_\alpha(t)dt \geq 0 \qquad (x>0),$$

for which the best possible range of parameters is known in an implicit formulation involving transcendental equations, and obtain the lower and upper bounds for this range of parameters. In addition, we apply our criteria to improve the positivity region for the Lommel functions established by J. Steinig in 1972.

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[2] Y.-K. Cho, S.-Y. Chung and H. Yun, *An extension of positivity for integrals of Bessel functions and Buhmann's radial basis functions*, Proc. Amer. Math. Soc., Series B, Vol. 5, pp. 25–39 (2018)
[3] Y.-K. Cho, S.-Y. Chung and H. Yun, *Rational extension of the Newton diagram for the positivity of ${}_1F_2$ hypergeometric functions and Askey-Szegő problem*, arXiv:1805.11855, Constr. Approx. (to appear) (2019)
[4] Y.-K. Cho and H. Yun, *Newton diagram of positivity for ${}_1F_2$ generalized hypergeometric functions*, Integral Transforms Spec. Funct. 29, pp. 527–542 (2018)