What is ... a Riemann-Hilbert problem?

Thomas Bothner
(*King's College London, UK*)
Time: Wednesday 24.07., 09:00, Room AM

Abstract: In its classical setting, the Riemann-Hilbert problem refers to Hilbert's 21st problem of constructing a Fuchsian ODE system with prescribed poles and a given monodromy group. Using singular integral equation techniques, Plemelj presented a solution to this problem in 1908 which became widely accepted. However, Kohn, Arnold and Il'yashenko noticed in the mid 1980s that Plemelj had actually worked on a problem similar to Hilbert's 21st for so-called regular ODE systems rather than Fuchsian ones. These new investigations resulted eventually in a negative answer to Hilbert's original problem given by Bolibruch in 1989 with further developments by Bolibruch and Kostov soon after.

Tangentially to the solution of Hilbert's classical problem, the singular integral equation techniques used therein, a.k.a. analytic factorizations of given functions defined on curves, gave rise to a class of modern Riemann-Hilbert factorization problems. In fact nowadays we view such problems as part of a broad analytical toolbox that is useful in the analysis of problems in mathematics and physics, for instance the Wiener-Hopf methods in hydrodynamics and diffraction. The goal of this talk is to first review some facts of the classical Riemann-Hilbert theory and then present a few recent developments of its modern counterpart. Special attention in the second part will be given to matrix- and operator-valued Riemann-Hilbert problems that arise in random matrix theory and integrable probability.