GCLC Prover Output for conjecture "thm-Ceva"

Wu's method used

July 10, 2009

1 Construction and prover internal state

Construction commands:

- Point A
- Point B
- Point C
- Point P
- Line a: B C
- Line b: A C
- Line c: A B
- Line pa: P A
- Line pb: P B
- Line pc: P C
- Intersection of lines, D: a pa
- Intersection of lines, E: b pb
- Intersection of lines, F: c pc

Coordinates assigned to the points:

- $A = (0, 0)$
- $B = (u_1, 0)$
- $C = (u_2, u_3)$
- $P = (u_4, u_5)$
- $D = (x_2, x_1)$
- $E = (x_4, x_3)$
- $F = (x_6, 0)$

Conjecture(s):

1. Given conjecture

    - **GCLC code:**

    ```
    equal { mult { mult { sratio A F F B } { sratio B D D C } } { sratio C E E A } } 1
    ```

    - **Expression:**
    $\left(\left(\frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}}\right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}}\right) = 1$

    - **Expression after rationalization:**
    $(((x(A) - x(F)) \cdot (y(B) - y(D))) \cdot (y(C) - y(E))) = (((1 \cdot (x(F) - x(B))) \cdot (y(D) - y(C))) \cdot (y(E) - y(A)))$

2 Resolving constructed lines

- $a \ni B, C, D$
- $b \ni A, C, E$
- $c \ni A, B, F$; line is horizontal (i.e., $y(A) = y(B)$)
- $pa \ni P, A, D$
- $pb \ni P, B, E$
- $pc \ni P, C, F$

3 Creating polynomials from hypotheses

- Point A
  no condition
- Point B
  no condition
- Point C
  no condition
- Point P
  no condition
- Line a: B C
  - point B is on the line (B, C)
    no condition
  - point C is on the line (B, C)
    no condition

- Line b: A C
  - point A is on the line (A, C)
    no condition
  - point C is on the line (A, C)
    no condition
- Line c: A B
  - point A is on the line (A, B)
    no condition
  - point B is on the line (A, B)
    no condition
- Line pa: P A
  - point P is on the line (P, A)
    no condition
  - point A is on the line (P, A)
    no condition
- Line pb: P B
  - point P is on the line (P, B)
    no condition
  - point B is on the line (P, B)
    no condition
- Line pc: P C
  - point P is on the line (P, C)
    no condition
  - point C is on the line (P, C)
    no condition
- Intersection of lines, D: a pa
  - point D is on the line (B, C)

$$p_1 = -u_3x_2 + (u_2 - u_1)x_1 + u_3u_1$$

  - point D is on the line (P, A)

$$p_2 = u_5x_2 - u_4x_1$$

- Intersection of lines, E: b pb
  - point E is on the line (A, C)

$$p_3 = -u_3x_4 + u_2x_3$$

- point E is on the line (P, B)

$$p_4 = u_5 x_4 + (-u_4 + u_1) x_3 - u_5 u_1$$

- Intersection of lines, F: *c pc*
  - point F is on the line (A, B) — true by the construction
    no condition
  - point F is on the line (P, C)

$$p_5 = (u_5 - u_3) x_6 + (-u_5 u_2 + u_4 u_3)$$

4 Creating polynomial from the conjecture

- Processing given conjecture(s).

Conjecture 1:

$$p_6 = -2 x_6 x_3 x_1 + u_3 x_6 x_3 + u_3 x_6 x_1 + u_1 x_3 x_1 - u_3 u_1 x_3$$

5 Invoking the theorem prover

The used proving method is Wu's method.

The input system is:

$$\begin{aligned}
p_0 &= -u_3 x_2 + (u_2 - u_1) x_1 + u_3 u_1 \\
p_1 &= u_5 x_2 - u_4 x_1 \\
p_2 &= -u_3 x_4 + u_2 x_3 \\
p_3 &= u_5 x_4 + (-u_4 + u_1) x_3 - u_5 u_1 \\
p_4 &= (u_5 - u_3) x_6 + (-u_5 u_2 + u_4 u_3)
\end{aligned}$$

5.1 Triangulation, step 1

Choosing variable: Trying the variable with index 6.

Variable x_6 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

5.2 Triangulation, step 2

Choosing variable: Trying the variable with index 5.

Choosing variable: Trying the variable with index 4.

Variable x_4 selected: The number of polynomials with this variable is 2.

Minimal degrees: 3 polynomials with degree 1 and 2 polynomials with degree 1.

Polynomial with linear degree: Removing variable x_4 from all other polynomials by reducing them with polynomial p_3.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_0 &= -u_3x_2 + (u_2 - u_1)x_1 + u_3u_1 \\ p_1 &= u_5x_2 - u_4x_1 \\ p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\ p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\ p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3) \end{aligned}$$

5.3 Triangulation, step 3

Choosing variable: Trying the variable with index 3.

Variable x_3 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

5.4 Triangulation, step 4

Choosing variable: Trying the variable with index 2.

Variable x_2 selected: The number of polynomials with this variable is 2.

Minimal degrees: 1 polynomials with degree 1 and 0 polynomials with degree 1.

Polynomial with linear degree: Removing variable x_2 from all other polynomials by reducing them with polynomial p_1.

Finished a triangulation step, the current system is:

$$\begin{aligned} p_0 &= (u_5u_2 - u_5u_1 - u_4u_3)x_1 + u_5u_3u_1 \\ p_1 &= u_5x_2 - u_4x_1 \\ p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\ p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\ p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3) \end{aligned}$$

5.5 Triangulation, step 5

Choosing variable: Trying the variable with index 1.

Variable x_1 selected: The number of polynomials with this variable is 1.

Single polynomial with chosen variable: No reduction needed.

The triangular system has not been changed.

The triangular system is:

$$\begin{aligned}
p_0 &= (u_5u_2 - u_5u_1 - u_4u_3)x_1 + u_5u_3u_1 \\
p_1 &= u_5x_2 - u_4x_1 \\
p_2 &= (u_5u_2 - u_4u_3 + u_3u_1)x_3 - u_5u_3u_1 \\
p_3 &= u_5x_4 + (-u_4 + u_1)x_3 - u_5u_1 \\
p_4 &= (u_5 - u_3)x_6 + (-u_5u_2 + u_4u_3)
\end{aligned}$$

6 Final remainder

6.1 Final remainder for conjecture 1

Calculating final remainder of the conclusion:

$$g = -2x_6x_3x_1 + u_3x_6x_3 + u_3x_6x_1 + u_1x_3x_1 - u_3u_1x_3$$

with respect to the triangular system.

1. Pseudo remainder with p_4 over variable x_6:

$$g = (-2u_5u_2 + u_5u_1 + 2u_4u_3 - u_3u_1)x_3x_1 +$$
$$(u_5u_3u_2 - u_5u_3u_1 - u_4u_3^2 + u_3^2u_1)x_3 + (u_5u_3u_2 - u_4u_3^2)x_1$$

2. Pseudo remainder with p_3 over variable x_4:

$$g = (-2u_5u_2 + u_5u_1 + 2u_4u_3 - u_3u_1)x_3x_1 +$$
$$(u_5u_3u_2 - u_5u_3u_1 - u_4u_3^2 + u_3^2u_1)x_3 + (u_5u_3u_2 - u_4u_3^2)x_1$$

3. Pseudo remainder with p_2 over variable x_3:

$$g = (u_5^2u_3u_2^2 - 2u_5^2u_3u_2u_1 + u_5^2u_3u_1^2 - 2u_5u_4u_3^2u_2 +$$
$$2u_5u_4u_3^2u_1 + u_5u_3^2u_2u_1 - u_5u_3^2u_1^2 + u_4^2u_3^3 - u_4u_3^3u_1)x_1 +$$
$$(u_5^2u_3^2u_2u_1 - u_5^2u_3^2u_1^2 - u_5u_4u_3^3u_1 + u_5u_3^3u_1^2)$$

4. Pseudo remainder with p_1 over variable x_2:

$$g = (u_5^2u_3u_2^2 - 2u_5^2u_3u_2u_1 + u_5^2u_3u_1^2 - 2u_5u_4u_3^2u_2 +$$
$$2u_5u_4u_3^2u_1 + u_5u_3^2u_2u_1 - u_5u_3^2u_1^2 + u_4^2u_3^3 - u_4u_3^3u_1)x_1 +$$
$$(u_5^2u_3^2u_2u_1 - u_5^2u_3^2u_1^2 - u_5u_4u_3^3u_1 + u_5u_3^3u_1^2)$$

5. Pseudo remainder with p_0 over variable x_1:

$$g = 0$$

7 Prover report

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions are:

- $P_{FBF} \neq 0$ i.e., points F and B are not identical (conjecture based assumption).
- $P_{DCD} \neq 0$ i.e., points D and C are not identical (conjecture based assumption).
- $P_{EAE} \neq 0$ i.e., points E and A are not identical (conjecture based assumption).