

Transformations & Inserting Pictures into the Graphics Window

GeoGebra Workshop Handout 4

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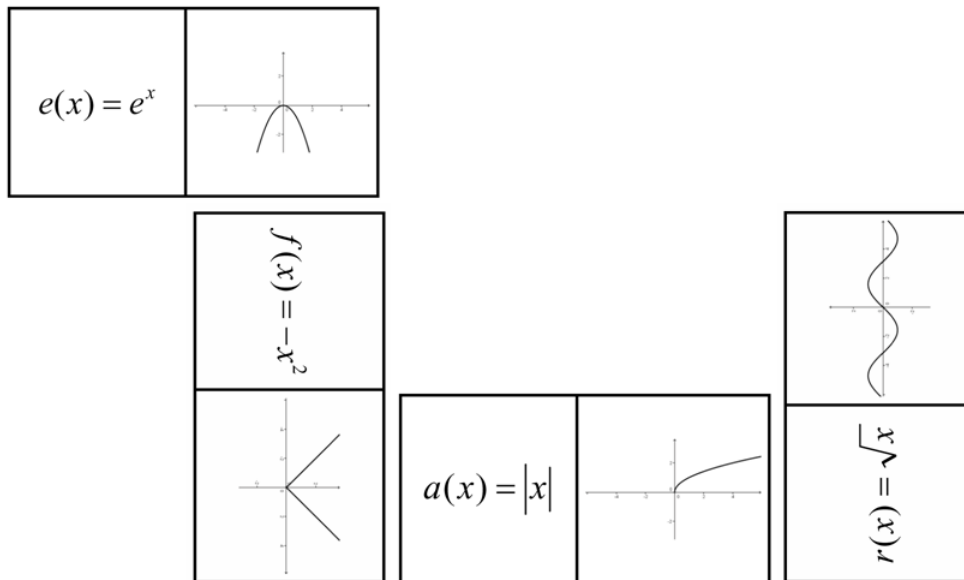
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1. Creating a 'Function Domino' Game


In this activity you are going to practice exporting function graphs to the clipboard and inserting them into a word processing document in order to create cards for a 'Function Domino' game.



Preparations

- Open a new GeoGebra document.
- Show the algebra window, input field, and coordinate axes (*View* menu).

Instructions for GeoGebra

1	Enter an arbitrary function. <u>Examples:</u> $e(x) = \exp(x)$ or $f(x) = \sin(x)$
2	 Move the function graph into the upper left corner of the drawing pad.
3	Reduce the size of the GeoGebra window so it only shows the desired part of the drawing pad.
4	Export the drawing pad to the clipboard. <u>Hint:</u> Menu <i>File – Export – Drawing Pad to Clipboard</i>



Instructions for MS Word

1	Open a new word processing document (e.g. MS Word).
2	Create a table with 2 columns and several rows. <u>Hint:</u> Menu <i>Table – Insert – Table...</i>
3	Highlight the entire table (all cells) and open the <i>Table Properties</i> dialog. <u>Hint:</u> Menu <i>Table – Table Properties...</i>
4	Click on tab <i>Row</i> and specify the row height as 2 inches.
5	Click on tab <i>Column</i> and set the preferred width of the columns to 2 inches.
6	Click on tab <i>Cell</i> and set the vertical alignment to <i>Center</i> .
7	Click the <i>OK</i> button.
8	Place the cursor in one of the table cells. Insert the function graph picture from the clipboard. <u>Hint:</u> Menu <i>Edit – Paste</i> or key combination <i>Ctrl – V</i>
9	Adjust the size of the picture if necessary. <u>Hint:</u> Double click the picture to open the <i>Format Picture</i> dialog window. Click on tab <i>Size</i> and set the longer side (either width or height) to 1.9".
10	Enter the equation of a different function into the cell next to the picture. <u>Hint:</u> You might want to use an equation editor.

Repeat the steps in GeoGebra for a different function (e.g. trigonometric, logarithmic) and insert the new picture into MS Word in order to create another domino card.

Hint: Make sure to put the equation and graph of each function on different domino cards.

2. Creating a ‘Geometric Figures Memory’ Game

In this activity you are going to practice exporting geometric figures to the clipboard and inserting them into a word processing document in order to create cards for a memory game with geometric figures. Make sure you know how to



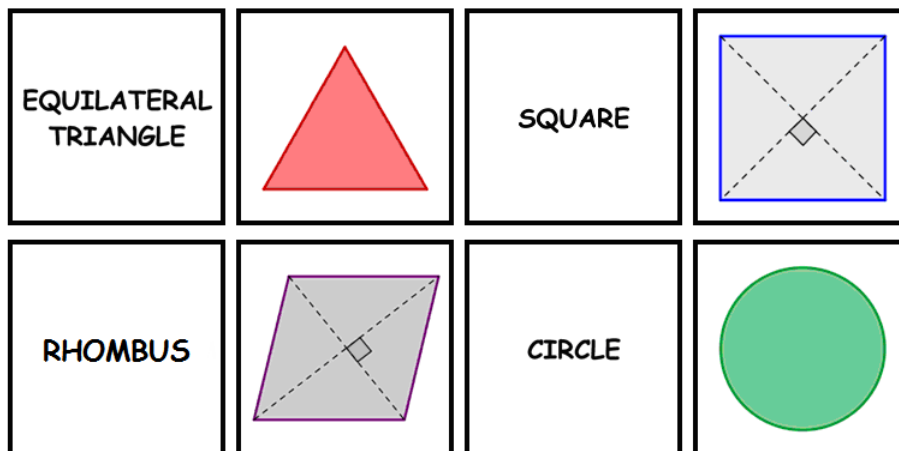
construct different geometric figures (e.g. quadrilaterals, triangles) before you begin with this activity.

Preparations

- Open a new GeoGebra document.
- Hide the algebra window, input field, and coordinate axes (*View menu*).

Instructions for GeoGebra

1	Construct a geometric figure in GeoGebra (e.g. isosceles triangle).
2	Use the <i>Properties dialog</i> to enhance your construction.
3	Move the figure into the upper left corner of the drawing pad and adjust the size of the GeoGebra window.
4	Export the drawing pad to the clipboard. <u>Hint:</u> Menu <i>File – Export – Drawing Pad to Clipboard</i>



Instructions for MS Word

1	Open a new word processing document (e.g. MS Word).
2	Create a table with 2 columns and several rows. <u>Hint:</u> Menu <i>Table – Insert – Table...</i>
3	Highlight the entire table (all cells) and open the <i>Table Properties</i> dialog. <u>Hint:</u> Menu <i>Table – Table Properties...</i>



4	Click on tab <i>Row</i> and specify the row height as 2 inches.
5	Click on tab <i>Column</i> and set the preferred width of the columns to 2 inches.
6	Click on tab <i>Cell</i> and set the vertical alignment to <i>Center</i> .
7	Click the <i>OK</i> button.
8	Place the cursor in one of the table cells. Insert the picture of the geometric figure from the clipboard. <u>Hint</u> : Menu <i>Edit</i> – <i>Paste</i> or key combination <i>Ctrl</i> – <i>V</i>
9	Adjust the size of the picture if necessary. <u>Hint</u> : Double click the picture to open the <i>Format Picture</i> dialog window. Then click on tab <i>Size</i> and set the longer side of the picture to 1.9 inches.
10	Enter the name of the geometric shape into another cell of the table.

Repeat the steps in GeoGebra for a different geometric shape (e.g. parallelogram, circle) and insert the new picture into MS Word in order to create another memory card.

Hint: Make sure to put the name and sketch of each geometric shape on one of the memory cards.

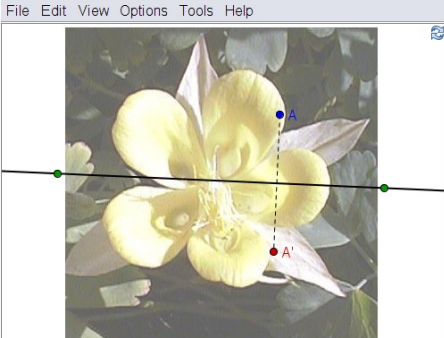
3. Exploring Symmetry with GeoGebra

Back to school...

Open the dynamic worksheet *04_exploring_symmetry.html*. Follow the directions on the worksheet and experience how your students could explore the axes of symmetry of a flower.

Axes of Symmetry

Below you can see a point **A** who was reflected at the line in order to create its image **A'**.





1. Drag point **A** with the mouse along the outline of the flower. What do you notice? Write down your **observations**.
2. How many **axes of symmetry** does this flower have?
Hint: Drag the **green points** in order to **change the position of the line** of reflection. Then, repeat step (1) for every position of the line.
Hint: Press the keys **Ctrl + F** at the same time in order to **delete the traces**.
3. Make a **sketch** of this worksheet including the flower and all lines of symmetry you were able to find.



Preparations










- Make sure you have the picture *05_flower.jpg* saved on your computer.
- Open a new GeoGebra file.
- Hide algebra window, input field, and coordinate axes (*View* menu).

Introduction of new tools

	Show / hide label	New!
	Mirror object at line	New!
<u>Hint:</u> Click the object to be mirrored and then click the line of reflection.		


Hints: Don't forget to read the toolbar help if you don't know how to use these tools. Try out the new tools before you start the construction.

Instructions

1		New point <i>A</i>
2		Show the label of point <i>A</i>
3		Line of reflection through two points
4		Mirror point <i>A</i> at line to get image <i>A'</i>
5		Segment between point <i>A</i> and its image <i>A'</i>
7		Turn the <i>Trace on</i> for points <i>A</i> and <i>A'</i> <u>Hint:</u> Right click (MacOS: <i>Ctrl</i> – click) the point and select <i>Trace on</i> from the menu. Whenever point <i>A</i> is moved it leaves a trace on the drawing pad.
8		Move point <i>A</i> to draw a dynamic figure
9		Insert the image <i>05_flower.jpg</i> into the drawing pad. <u>Hint:</u> Click in the lower left corner of the drawing pad to insert the picture at this position.
10		Adjust the position of the inserted image.
11		Set the image as <i>background image</i> (<i>Properties dialog</i> , tab <i>Basic</i>).
12		Reduce the <i>filling</i> of the image (<i>Properties dialog</i> , tab <i>Style</i>).



Hint: After specifying the picture as a background image you need to open the *Properties dialog* using the *Edit* menu. You can't select a background image in the graphics window any more.

Hint: The  *Trace on* feature has some special characteristics:

- The trace is a temporary phenomenon. Whenever the graphics are refreshed, the trace disappears.
- The trace can't be saved and is not shown in the algebra window.
- To delete the trace you need to refresh the views (menu *View – Refresh views* or key combination *Ctrl – F*. MacOS: *Open Apple – F*).



4. Resizing, Reflecting, and Distorting a Picture



In this activity you will learn how to resize an inserted picture to a certain size and how to apply transformations to the picture in GeoGebra.

Preparations



- Make sure you have the picture *06_Palmtrees.jpg* saved on your computer.
- Open a new GeoGebra file.
- Close the algebra window and hide the coordinate axes (*View* menu).



Instructions for reflecting and resizing a picture

1		Insert picture <i>06_Palmtrees.jpg</i> into the left part of the drawing pad
2		Point A at the lower left corner of the picture
3		Set point A as the FIRST corner point of your picture. <u>Hint:</u> Open the <i>Properties dialog</i> and select the picture in the list of objects. Click on tab 'Position' and select point A from the drop-down list next to <i>Corner 1</i> .
4		Point B = A + (3, 0)
5		Set point B as the SECOND corner point of the picture. <u>Hint:</u> You just changed the width of the picture to 3 cm.






6		Line through two points in the middle of the drawing pad
7		Mirror the picture at the line <u>Hint</u> : You might want to reduce the filling of the image in order to be able to better distinguish it from the original (<i>Properties dialog</i>).

Back to school...

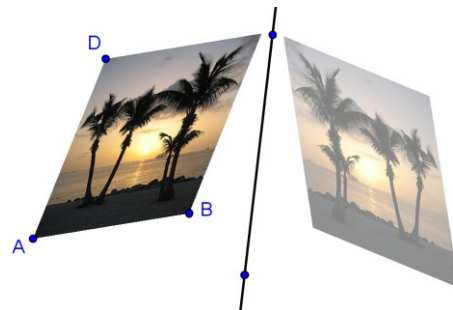
- (a) Move point *A* with the mouse. How does this affect the picture?
- (b) Move the picture with the mouse and observe how this affects its image.
- (c) Move the line of reflection by dragging the two points with the mouse. How does this affect the image?

Instructions for distorting a picture

1		Open the figure you created in the previous activity.
2		Delete point <i>B</i> to restore the picture's original size
3		Create a new point <i>B</i> at the lower right corner of the original picture.
4		Set the new point <i>B</i> as the second corner point of your picture. <u>Hint</u> : You can now resize the image by moving point <i>B</i> .
5		Create a new point <i>D</i> at the upper left corner of the original picture. <u>Hint</u> : GeoGebra offers a 'fast-renaming' option. Activate <i>Move</i> mode and select the object. When you start typing the new name GeoGebra opens the <i>Rename</i> dialog window.
6		Set the new point <i>D</i> as the FOURTH corner point of your picture

Back to school...

- (a) How does moving point *D* affect the picture and its image?
- (b) Which geometric shape do the picture and the image form at any time?



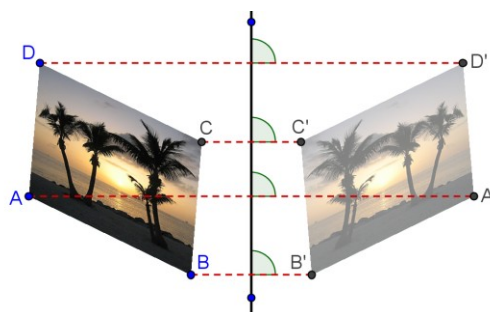


5. Exploring Properties of Reflection

In this activity you will create a dynamic figure that allows your students to explore the properties of reflection.

Preparations

You will now modify the construction created in the previous activity. If you want to keep the original as well you need to save your file.



Instructions

1		Open the file you created in the previous activity which contains the distorted picture of the palm trees and its reflection at a line.
2		Segment between points A and B
3		Segment between points A and D
4		Parallel line to segment AB through point D
5		Parallel line to segment AD through point B
6		Intersect the two lines to get intersection point C
7		Hide auxiliary objects
8		Reflect all four corner points A , B , C , and D at the line to get their images A' , B' , C' , and D' .
9		Connect corresponding points with segments (e.g. points A and A')
10		Create angles between the line of reflection and the segments

Back to school...

(a) Move the corner points A , B , C , and D of the original picture. Are you able to drag all these points with the mouse? If no, which one can't be dragged and why?

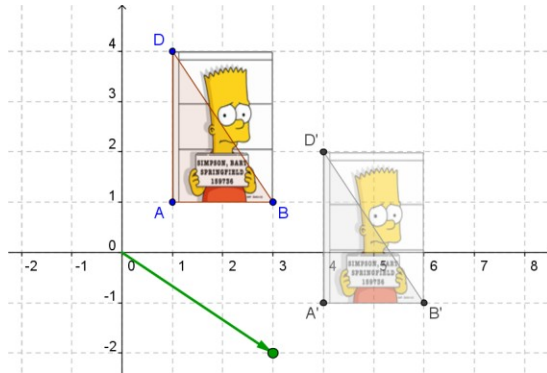
(b) Move the line of reflection. What do you notice about the angles between the segments connecting the corresponding corner points and the line of reflection?



6. Translating Pictures

Preparations

- Make sure you have the picture *07_Bart.png* saved on your computer.
- Open a new GeoGebra file.
- Show the algebra window, input field, coordinate axes, and grid (*View* menu).
- In the *Options* menu set the point capturing to *on (Grid)*.



Introduction of new tools





	Vector between two points New!
<u>Hint:</u> First click determines the starting point and second click sets the vector's endpoint.	
	Translate object by vector New!
<u>Hint:</u> Click the object to be translated and then click the translation vector.	

Hints: Don't forget to read the toolbar help if you don't know how to use these tools. Try out the new tools before you start the construction.

Instructions

1		Insert picture <i>07_Bart.png</i> into the first quadrant.
2		Points $A = (1, 1)$, $B = (3, 1)$, and $D = (1, 4)$
3		Set point <i>A</i> as the FIRST, <i>B</i> as the SECOND, and <i>D</i> as the FOURTH corner point of the picture. <u>Hint:</u> <i>Properties dialog</i> – tab <i>Position</i>
4		Triangle <i>ABD</i>
5		Points $O = (0, 0)$ and $P = (3, -2)$
6		Vector $u = \text{Vector}[O, P]$ <u>Hint:</u> You could also use tool <i>Vector between two points</i> .




7		Translate the picture by vector u . <u>Hint</u> : You might want to reduce the filling of the image.
8		Translate the three corner points A , B , and D by vector u .
9		Triangle $A'B'D'$
10		Hide point O so it can't be moved accidentally.
11		Change the color and size of objects to enhance your construction.

7. Rotating Polygons

Preparations




- Open a new GeoGebra file.
- Hide algebra window and input field (*View* menu).
- Show the coordinate axes and the grid (*View* menu).
- Open the *Properties dialog for drawing pad* by right clicking the drawing pad (MacOS: *Ctrl* – click).
 - On tab *Axes* – *xAxis* change the *Distance* for x-Axis to 1
 - On tab *Axes* – *yAxis* change the *Distance* for y-Axis to 1

Introduction of new tool





	Rotate object around point by angle New! <u>Hint</u> : Click the object to be rotated, the center of rotation, and enter the angle in the appearing dialog window.
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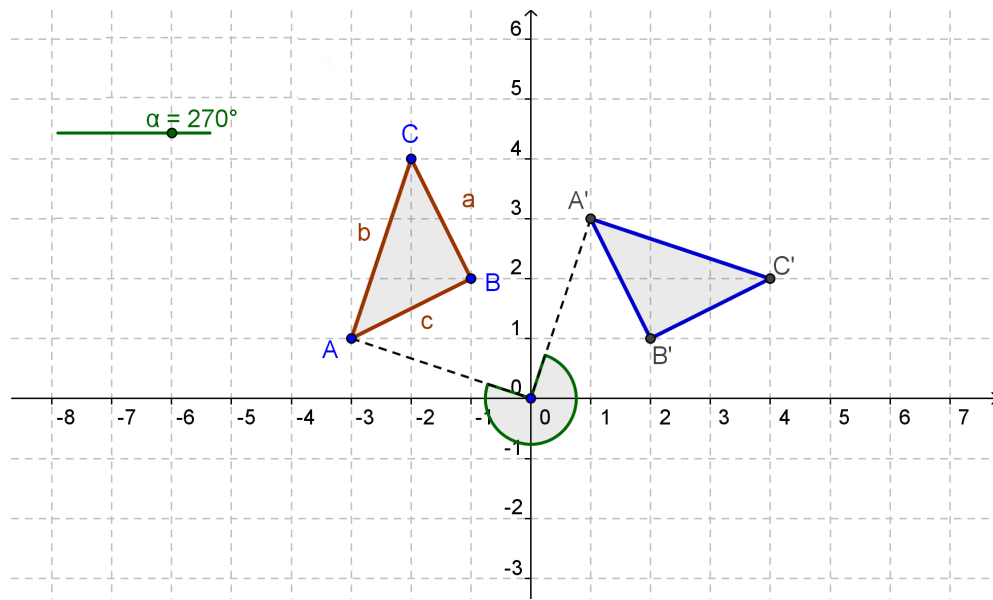
Hints: Don't forget to read the toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Instructions

1		Create an arbitrary triangle ABC in the second quadrant placing the vertices on grid points.
2		New point D at the origin of the coordinate system
3		Rename the new point to O <u>Hint</u> : GeoGebra offers a 'fast-renaming' option. Activate <i>Move</i> mode and select the object. When you start typing the new name GeoGebra opens the <i>Rename</i> dialog window.



4		Slider for angle α <u>Hint:</u> In the slider dialog window check <i>Angle</i> and set the increment to 90° . Make sure you don't delete the $^\circ$ symbol.
5		Rotate triangle ABC around point O by angle α <u>Hint:</u> Check <i>counter clockwise</i> rotation.
7		Segments AO and $A'O$
8		Angle AOA' <u>Hint:</u> Select the points in counter clockwise order. Hide the label of this angle.



Enhancing the construction

You will now learn how to 'tidy up' the algebra window by defining some objects as 'Auxiliary objects' and hiding their algebraic representation from view.

- Show the *Algebra window*.
- Open the *Properties dialog*.
- Select all segments in the *Properties dialog* and check *Auxiliary object* on tab 'Basic'.

Hint: Click on the heading 'Segment' in order to select all segments.

- Repeat this step for the triangles, angles, and point O at the origin.
Hint: The algebra window now only contains points A , B , and C as well as their images A' , B' , and C' .
- Make sure the item *Auxiliary objects* is unchecked in the *View menu*.



Note: Your students can now check out the coordinates of the initial points and their images in the algebra window without being distracted by the algebraic representation of the other objects used in this construction.

8. Challenge of the Day: Tilings with Regular Polygons

Go to folder *08_Tilings* and open the dynamic worksheet *01_tilings_triangle.html*. It is the first one of a series of ten dynamic worksheets that form a learning environment to explore tilings with regular polygons.

Hint: Use the links “[previous](#) | [back](#)” in the upper right corner of the dynamic worksheets to navigate in this learning environment.

Back to school...

(a) Work through the tasks on the dynamic worksheets of this learning environment. Write down your answers on paper and discuss them with colleagues afterwards.

(b) After working through the dynamic worksheets you should be able to answer the following questions:

- Which regular polygons can be used to tile the plane?
- Which transformation(s) did you use for the tilings?
- How many of each of these polygons meet at one of their edges?

(c) Fill in the missing values in the table below. Can you see any patterns? Try to find the formulas for an n -sided polygon.

polygon			parts		polygon
# vertices	tiling possible yes / no	# meet	central angle	interior angles	interior angles
3			---	---	
4					
5					
6					
7					
...
n					

(d) Come up with a conjecture that helps you to reason why not every regular polygon can be used for tilings.



Tilings with regular polygons – worksheet solution

polygon			parts		polygon
# of vertices	tiling possible yes / no	# for tiling	central angle	interior angles	interior angles
3	yes	6	---	---	60°
4	yes	4	$\frac{360^\circ}{4} = 90^\circ$	$\frac{180^\circ - 90^\circ}{2} = 45^\circ$	$2 \cdot 45^\circ = 90^\circ$
5	no	---	$\frac{360^\circ}{5} = 72^\circ$	$\frac{180^\circ - 72^\circ}{2} = 54^\circ$	$2 \cdot 54^\circ = 108^\circ$
6	yes	3	$\frac{360^\circ}{6} = 60^\circ$	$\frac{180^\circ - 60^\circ}{2} = 60^\circ$	$2 \cdot 60^\circ = 120^\circ$
7	no	---	$\frac{360^\circ}{7} \approx 51\frac{3}{7}^\circ$	$\frac{180^\circ - 51\frac{3}{7}^\circ}{2} \approx 64\frac{2}{7}^\circ$	$2 \cdot 64\frac{2}{7}^\circ = 128\frac{4}{7}^\circ$
...
n	no for $n > 6$	---	$\frac{360^\circ}{n}$	$\frac{180^\circ - \frac{360^\circ}{n}}{2}$	$180^\circ - \frac{360^\circ}{n}$