CADGME-2009 Abstracts

Keynote Addresses

Paul Andrews- University of Cambridge, United Kingdom

Title	Understanding the cultural dimension in research in mathematics teaching and learning

Abstract

Recent research has highlighted the significance of a teacher's cultural location as a determinant not only of his or her construal of mathematics but also of the didactic strategies he or she employs. However, the majority of research into the teaching and learning of mathematics assumes not only that mathematics as a body of knowledge and processes is cross-culturally constant but also that research undertaken in one cultural context has a transparent and unproblematic resonance with another. This assumption permeates all aspects of mathematics education research, and research into the use of technology to facilitate mathematical learning is no exception. In my talk I will highlight some of the ways in which culture influences education in general and mathematics education, including technologyenhanced mathematics education, in particular, with a view to explicating some of the factors that researchers in the field may need to acknowledge in their work.

Ralph - Johan Back- Åbo Akademi University, Turku, Finland

Title	Structured derivations: A method for Teaching proofs in High school mathematics

Abstract

Mathematics is based on proofs. A theorem without a proof is like magic, a theorem with a proof is self-evident (although it sometimes requires quite a lot of effort to see this). But proofs are considered difficult in today's high school (age group 16 -19) mathematics and are therefore usually avoided. When proofs are given, they are often informal and the underlying logic is not explicated.
Mathematical proofs are based on logic, but the use of logic in proofs is usually not taught in high schools today. Where logic is taught, it is seen as a separate object of study, rather than a tool to be used when solving mathematical problems.
Structured derivations, originally developed by J. von Wright and me in 1998 as an extension of E. W. Dijkstra's calculational proof method, provides an alternative approach to teaching mathematics in high school. It is based on a fixed logic based format for presenting proofs, derivations and calculations in a unified manner. The fixed format and the detailed justification of each proof step makes it is easier to

explain proofs in class. Structured derivations also makes it easier for students to understand the mathematical argument in the proof, helps them in constructing their own proofs and to detect errors in their proofs.
The talk will provide an overview of structured derivations, as well as describe the main results of our teaching experiments. Our research group has done quite a lot of experiments and empirical studies of structured derivations in high school mathematics, with very encouraging results. The method has been extended and refined during the last five years in a tight feedback loop with experiments where the method has been tested in practice. The method is accepted by the Finnish National Board of Education, and is presently taught in our continuous education courses for mathematics teachers in high school.

Bruno Buchberger- Research Institute for Symbolic Computation
Johannes Kepler University, Linz / Hagenberg, Austria

Title	**Mathematical Invention: How Much Can be Automated?**

Abstract

Inventing mathematical knowledge (theorems and methods), in interaction with proving and disproving, is the essence of mathematical creative activity. How much of this can be automated?

We argue that, paradoxically, automating the creative activity of mathematics is, ultimately, the goal of mathematics: Higher layers of creativity automate previous layers of creativity. The spiral of going through higher and higher levels of automation has no upper bound.

This view is intimately connected with teaching mathematics: Teaching is essentially explanation, i.e. "making seemingly difficult things plain". This is possible by deeper insight, "seeing and finding" the keys in seemingly complicated situations. A complete insight may typically lead to a method for inventing certain mathematical facts.

Automation of creative mathematical invention can be observed already in the early days of mathematics. It became more an more pronounced through the history of mathematics. Through the advent of computers, this automation is accelerating in present time. Teaching should follow this path from "creative" consideration of individual instances to an automated procedure.

In the talk, we will give a couple of examples of automating mathematical invention by (meta-)mathematical means. We start from simple algorithms that - in an appropriate perspective - may be viewed as automated invention methods. We proceed to considering developments in computer algebra of the past four decades, for example Wu's method, that may also be interpreted as - powerful - invention methods. Finally, we will explain two recent methods proposed by the speaker that allow to invent mathematical theorems and algorithms in the context of computer-supporting mathematical theory exploration. One of the methods is "scheme-based" invention that has a lot to do with the notion of "functors", the second method is a top-down method that systematizes the approach of "learning from failure".

André Heck- University of Amsterdam, Netherlands

Title	**Modelling in Cross-Disciplinary Authentic Student Research Projects**

Abstract

In the Dutch secondary education system students are obliged to carry out at the end of their school career their own research project of approximately 80 hours. They are stimulated to choose the topic themselves (preferably with relations to two subjects, like mathematics and physics, or mathematics and biology) and have a lot of freedom in the design of the research. Ideally students experience the stimulating aspects of doing research, and this is especially the case when the project is advanced and authentic, and even comparable with actual research done by real researchers. With the help of ICT (for data acquisition, video-analysis, modelling, data-analysis) the level and methods of the work of the students becomes quite close to the type of work of researchers in the field.

Since five years, mathematics and science freshmen at the University of Amsterdam do a small mathematical project in Calculus 1 using *Mathematica*. Once more a strong motive is to let students experience at an early stage of their university career that a rather basic knowledge of mathematics and science in combination with the use of sophisticated tools already allows them to do independent problem-solving and to successfully carry out a mathematical investigation.

In this talk, examples of students' work about human gait, intake and clearance of alcohol in the human body, bungee jumping, and chaotic motion will be presented and the role of technology in mathematical modelling will be discussed.

Jozef Hvorecky, Vysoka skola manazmentu, Bratislava, Slovakia

University of Liverpool, Liverpool, United Kingdom

Title	Using a Managerial Analogy for Making Mathematics More Attractive

Abstract

Mathematics does not belong among the most popular subjects. At the same time, its importance for knowledge society is undisputable. For that reason it is rather critical to invite the entire community to participate in the process of its improvement. There are two principal ways to changing the *status quo*:

- The first („internal“) one is based on improving the quality of its teaching. This approach is a frequent subject of mathematical research.
- The second („external“) one exploits an analogy with business. As business persistency strongly depends on the ability of its management to promote accountability and responsiveness to the community, opening channels for information exchange with their environment is highly valued. Research targeting this approach is much less frequent. In our paper, we propose intensifying „marketing of mathematics“ i.e. inviting people to share their vis matics.

We will commence our presentation by showing that not all elements of Zimmerman's octagon [1] are covered equally by current mathematics courses. Some of them (e.g. *Calculate* and *Construct*) are dominating; others (namely *Play* and *Argue*) appear in the curriculum exceptionally. As all eight elements

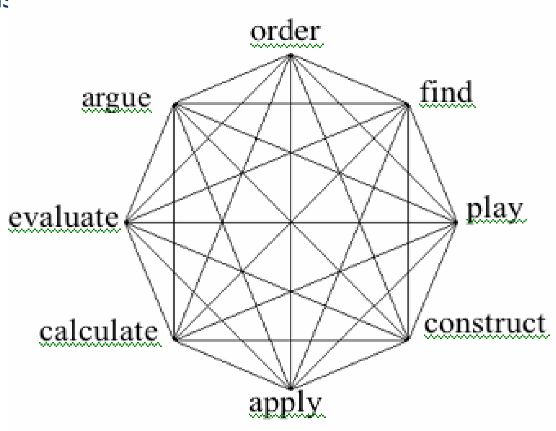

Figure 1. Zimmerman's octagon

represent basic mathematical skills, they should be present in a more balanced manner. *Play* and *Argue* can be characterized as "soft skills" which may demonstrate the usefulness of Mathematics to much wider audience than the "hard ones" – *Calculate* and *Construct*. In our presentation, we will demonstrate potential benefits of the "soft skills" and concentrate on the issues as: *Why should we make Mathematics Education more attractive? How to do it without weakening its quality? How to explain its strengths and weaknesses in a popular language and why? Can information technology help us and, if so, how?*

Reference:

1. Zimmermann, B. 2003. **On the Genesis of Mathematics and Mathematical Thinking - a Network of Motives and Activities Drawn from the History of Mathematics**. In L. Haapasalo and K. Sormunen (Eds.) *Towards Meaningful Mathematics and Science Education*. University of Joensuu. Bulletins of the Faculty of Education 86, 29-47.

Contributed Talks

Name	**Celina Abar**
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Title	**Virtual Learning Environment and others Technologies used in Continuous Formation of Mathematics Teachers**

Abstract

This work presents the results achieved in continuous training of mathematics teachers in the Post-Graduate Program Studies in Mathematic Education, with the use of a virtual learning environment (VLE) Moodle and others technologies. It is difficult to conceive a teacher, nowadays, developing his work at school without any technological apparatus to support his activities. Thus teacher's training must be continuous and it has to progress according to technologic developments. It is essential to know not only the strategies of teaching and learning with the use of specific programs that allows the development of mathematical content, but also the theoretical basis which gives support to such strategies as a structure for teacher's training. The use of the environment Moodle allowed us to break barriers of time and distance; it was an interactive and collaborative work with very positive results. It is gratifying to notice these results, because even those teachers reluctant in using computers become enchanted with their own abilities' progress. The participants get involved in the learning's context; they propose problems and its resolutions, formulate hypotheses, observe and review their previous designs, make decisions, in a dialogue with the reality of their practice teaching. The specific objective of the continuous formation of these students-teachers meets the following goal: develop a training supported by the knowledge's appropriation of research in Information and Communication Technologies (ICT) and their interface with Mathematics Education. Thus it is expected to lead student-teacher to recognize the possibilities of technologies to their professional practice and identify procedures, concepts, descriptions and representations which may be useful. To this end, specific modules are scheduled in which the student must act individually and collectively to achieve the objectives proposed. The programmatic content has been structured having in view the analysis and proposals to use softwares

as support to teaching and learning of mathematics and the discussion of texts and theoretical research mathematical content developed suppose an practical training. Activities with content on functions, geometry and algebra were proposed with the use of WIRIS Cas and Geogebra. In the studies of Dynamic Geometry the programs Cabri Géomètre and Geogebra were chosen because they're used in most schools and also because Geogebra permit free access and a practical use. It´s important to take into account in the designing of the course, the pedagogic proposal of the discipline and the analysis of the possibilities of interaction that allow the teacher to monitor the development of students individually and in groups, their expectations and their reflections on the construction of their knowledge. The guiding principle is to allow the collaborative learning and the knowledge's sharing, because we believe that learning is not a solitary process. All the subjects proposed had as support texts and theoretical research on the use of ICT in teaching and learning of mathematical content to subsidize the development of the training.

Name	**Steve Arnold**
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Title	

Abstract

While we are seeing a growing range of software options which offer both computer algebra systems (CAS) and interactive or dynamic geometry systems (DGS) within the one envelope, in most cases these appear not to offer a "true" integration of these two environments: rather, the systems exist alongside each other with some nominal interaction. This session considers some examples of integration currently available, including GeoGebra, TI-Nspire CAS, Geometry Expressions and the new MathRider. It explores applications of such integration to the teaching and learning of mathematics, particularly at high school level, and finally poses questions regarding the possibilities for future development towards true dynamic integration between CAS and DGS.

Name	**A. Homero Flores**
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Title	**Learning Math, Doing Math: Deductive Thinking and Construction Tasks with The Geometer's Sketchpad**

Abstract

A deductive thinking can be considered as the concatenation of ideas, each one determined by the previous one. In mathematics, deduction is the way in which we validate a conjecture, using general facts to justify less general or particular facts.
All individuals possess this way of constructing thoughts and jumping into conclusions, but we need to practice it in order to develop it deeper and get conclusion more educated, and so the decision we make will be the best ones at that moment.
In construction tasks with The Geometer's Sketchpad it is possible to explore the construction itself, make conjectures and try to validate them. When we try to explain why a construction works or why a conjecture is valid, we put ourselves in a theoretical context and make use of deduction as the means of getting the right answers.
When students work on a mathematical problem, they are used to think in the final result, not in the process they followed to get that final result. Most of the time, is in these processes were students find the justification of a conjecture and a review of the knowledge they already have.
In Learning Math, Doing Math, I propose, among many things, the fostering of a deductive thinking mainly – but not exclusively- with the development of construction tasks in Euclidian Geometry. An important part of the construction tasks is to ask students to explain how they did it and what suppositions they made and why. Now, the drag function of Sketchpad is of great help in exploring constructions in search of the right construction and validation of conjectures.
In this workshop, aimed at high school teachers and people interested in deductive thinking, we will go

through the next activities to illustrate what I said in the previous paragraphs. They are a good way to introduce on the software and the teaching model activities. It is not mandatory that attendees have Sketchpad experience.
Activity 1
Construct a square using three different ways of doing it. In each case, explain why your construction is a square.
Actvity 2
Which kind of quadrilateral is the one that its diagonals meet each other at their midpoints? Explain your answer.
Construct the quadrilateral with Sketchpad. Write down step by step how the construction was made.

Name	**Christian Bokhove**
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Title	**Assessing symbol sense in a digital tool**

Abstract

My research combines three key topics within the scope of the CADGME conference: algebraic skills, instrumentation, and assessment in digital environments. The conceptual framework behind the research contends that these three topics have to be integrated into one to provide a clear picture, as they are interlinked. Instrumentation of ICT tools influences the way algebra is learned and vice versa. The use of ICT tools influence assessment practices. The requirements of algebra determine how we assess algebra. Based on several externally validated criteria for ICT tools for assessing algebraic skills (reference) we implemented a digital tool in the Digital Mathematics Environment that:
- Assesses both basic skills and symbol sense;
- Provides an open environment and feedback to facilitate formative assessment;
- Stores both answers and the solution process of the student;
This tool was deployed in seven two-hour 1-to-1 sessions with students from secondary school. The data – audio and video/screencam footage- was analyzed with Atlas TI. Key observations concerned occurrences of symbol sense, and misconceptions of algebra. For the second revision one central question was how to overcome these obstacles by the tool providing feedback. These results were used for a second revision of the prototypical tool. This revision will be used for experiments on a larger scale with a web-based tool for acquiring, practicing and assessing algebraic skills.
This presentation shows examples of (lack of) symbol sense in the 1-to-1 sessions, showing that demanding the right features from a software tool, enables students to formatively assess conceptual understanding. We will also link the results to design requirements for the revision, paying particular attention to (the types of) feedback that could be provided in this revision.

Name	**Francisco Pérez-Arribas**
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Title	**Teaching Computer-Aided Design with the use of Dynamic Geometry**

Abstract

This paper describes the teaching of geometry at university level, more precisely at an engineering university where practical applications are important. In this particular example, a Dynamic Geometry methodology is used to teach the foundations of Computer Aided Design (CAD) to students of 4th year (out of 5 years of university education).

CAD began with the use of plane curves in design during the 1960's, employing graphical algorithms that are the base for numerical ones. Those graphical algorithms worked well because of projective geometry and projective properties of conics discovered in the XVII century.

The use of dynamic geometry allows a better understanding of the CAD basis, how and why it works, and a revision of the student's background knowledge in projective geometry with the use of modern and interactive computer techniques.

Dynamic Geometry is very appropriate in this case because it allows both the graphical and the numerical definition. Some algorithms are described graphically and then reproduced numerically with the use of algebra, in the same Dynamic Geometry environment.

Even though the Dynamic Geometry program is not used as a CAD program or as a programming environment, it is a good joint of both styles and sets a good base to start using a CAD program or programming CAD algorithms that are later used by the students in their subject program and in their future professional work. The understanding of the CAD algorithms is important to use the potential of CAD programs and understand what the program is doing in order to avoid mistakes.

Name	**Peter Samuels**
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Title	**Design of a Mobile Mathematics Creativity Laboratory for Contemporary Learners**

Abstract

Many developed countries have a serious problem with the engagement and future career choice of secondary and tertiary level students with mathematics learning leading to a crisis in the supply of mathematically trained graduates. The way in which mathematics is taught at the post compulsory level has generally not kept pace with major cultural changes affecting contemporary learners, in particular, the emergence of the digital native generation, the increasing prevalence of personal data and sensing devices amongst students and the emergence of the web 2.0 paradigm of internet content sharing. Furthermore, there is evidence that providing an enjoyable experience of mathematics for students can lead to an affective change in their attitude towards the subject which can then improve engagement, learning and future participation with the subject.

This presentation will describe the design of an enjoyable student-led environment for developing collaborative student-led mathematics projects comprising of mobile devices, LEGO Mindstorms NXT Robots, GeoGebra and visualisation software. The aim of the laboratory is to provide an environment which they can easily relate to that allows them to play with mathematical concepts by constructing, collaborating and sharing ideas. The connection between the design of the laboratory and the exploitation of ideas from the design of serious games will be described. In particular, the educational effectiveness of students designing games rather than simply playing them will be explored. The facilitation of student-led collaborative projects within the laboratory will be described along with current work with implementing the laboratory concept.

Conclusions area drawn about the future way in which mathematics might be taught and the possible integration of web 2.0 compliant computer algebra and dynamic geometry software, such as GeoGebra, with other technologies which can assist mathematics learning and affect for contemporary learners.

Name	**Andreas Fest**
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Title	**Creating interactive User Feedback in DGS using the Scripting Interface of Cinderella**

Abstract

Feedback is an importent component of interactive learning software. A conclusion from cognitive learning theory is that good software must give the learner more information about what he did. Following the ideas of constructivist learning theory the user be in control of both the time and the level of feedback he recieves. At the same time the feedback system must identify and review different possible solution strategies in an open learning environment.

The interactive geometry software Cinderella offers an easy-to-use programming interface based on the built-in scripting language CindyScript. This interface can be used to implement application specific feedback by the author of learning units.

In this talk we present two example learning units implementing two kinds of interactive feedback: feedback on demand and immediate feedback. The presented units come from discrete mathematics and from the theory of line reflections and congruencies in geometry. The units are implemented in a process-oriented design. Our activities support both the learning of the students and analysing the students' processes of learning mathematics.

Various directly given or hidden hints help the students to understand the mathematical principles behind the given problems. Our tools analyses the student's solution processes automatically and generates feedback on demand to the students.

The second learning environment can also be used in conjunction with recording of user actions. This allows additional feedback given later by the teacher whenever the automatic feedback system fails in analysing the users' learning processes. First experiences using the units in teaching are presented.

Name	**Judith Hohenwarter and Markus Hohenwarter**
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Title	**Introducing Dynamic Mathematics Software to Teachers: the Case of GeoGebra**

Abstract

In this talk we will describe a study aimed to identify most common impediments related to the introduction of the open-source mathematics software GeoGebra with the goal to facilitate the first contact of teachers with this software package in the future. We will report on the analysis of data collected during a two-week professional development programme that was organised for middle and high school teachers in Florida. The study identified challenges participants face during professional development workshops with technology and evaluated the difficulty levels of GeoGebra's construction tools. Findings of the study, complexity criteria for dynamic geometry tools, and common difficulties occurring in introductory technology workshops, provided the basis for the development of several new workshop materials that assist workshop activities and contributed to the improvement of GeoGebra introductory workshops for secondary school teachers.

Name	**Markus Hohenwarter**
Affiliation	Florida State University, United States
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Title	**GeoGebra - Past, Present, and Future of Dynamic Mathematics Software**

Abstract

The free dynamic mathematics software GeoGebra was originally based on ideas from interactive geometry and algebra systems and aimed to dynamically link them in one easy-to-use software package. GeoGebra's latest version now also includes a dynamic spreadsheet component and allows dealing with basic statistics concepts. Future versions plan to include symbolic algebra and 3D graphics extensions as well. These developments are driven by volunteers from all around the world, trying to give students and educators easy access to dynamic mathematics software. In this presentation, I will discuss the emergence of GeoGebra and its open-source user and developer community, provide examples for applications, and dare a glimpse into the future of such dynamic mathematics software. With this presentation, I also hope to nurture future cooperation with other open-source initiatives interested in aspects of converging mathematics assistants.

Name	**Natalija Budinski**
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Title	**Learning and teaching mathematics in high school trough real life models**

Abstract

In this paper we discuss the use of mathematical modeling as a tool for learning mathematics in high school. All problems are connected with exponential function. Students are encouraged to use computers and educational software as GeoGebra to present their task achievements. We also describe an example of model eliciting activities in the classroom. The teaching process is consisted of two parts. The group workshops and homework projects. Firstly, students are given some real situations and problems in classes. Their task is to make a model of the problem. The designed model needs to be creative and presented to fellow students. The second part is homework projects given to the students. Their task is to make a kind of research using internet and educational software in aim to find a mathematical model. The main purpose of this paper is to present the importance of modeling teaching in high school mathematics. Also, it is very important to teach students how to connect real life situations and mathematics. Teacher role is to guide students in their projects. This paper concludes student's views about mathematical modeling.

Name	**Yves Kreis, Carole Dording**
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Title	**GeoGebraPrim – GeoGebra for Primary School**

Abstract

The aims of the project are on one hand the improvement of the children's understanding of the elementary geometric concepts and on the other side a deeper insight in the bond of geometry and algebra.

The study consists in a pre-/post-test scenario combined with observations of particularly interesting situations. All in all approximately 200 children of 9 years will participate; about half of them will follow a traditional paper/pencil geometry course while the others will be mixing experiences on the computer to the traditional ones.

So far the existing Dynamic Geometry System GeoGebra has been adapted to the needs of the primary school through multiple observations of children working with the software. Besides it has been integrated into the TAO (Testing Assisté par Ordinateur – Computer-Assisted Testing) platform that will be used partially for the final test.

In this presentation we will give a short overview of the project, explain the educational and technological concepts used and present first results of the study.

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Title	**Remote symbolic computation of loci**

Abstract

In general, a locus is a set of points determined by some condition. When working with a geometric configuration in a Dynamic Geometry System (DGS), given a point T, dependent on the point M, which in turn is a point on a one-dimensional object l (line, circle,...), the locus defined by T and M is the set of points traced by T as M moves along l. The points T and M are generally referred to as the "tracer" and the "mover" of the locus respectively.

Standard DGS come equipped with functions that produce a graphical representation of a locus as the plotting of a sample of instances of its tracer point. Moreover, some DGS, such as Cabri, use the coordinates of some sample points to numerically produce equations of some of these loci. However both graphs and equations of loci produced with these methods are prone to inaccuracies .

This talk will present a web-based tool designed to compute exact equations and graphs of geometric loci specified using the commercial DGS Cabri Geometry II plus, The Geometer's Sketchpad and Cinderella2.0.

The equations produced by the application are remotely computed using symbolic algebraic techniques from the field of Automated Deduction based on Gröbner bases.

The tool, named LADucation, is an educational oriented web application with a simple interface in which the interaction with the user has been reduced to just one click.

Besides computing geometric loci, in the case of a Cabri geometric construction, LADucation also gives "certified" answers to the five questions considered by the Cabri in-menu property checker: collinear? Parallel? Perpendicular? Equidistant? Member?

The tool is freely available at http://nash.sip.ucm.es/LAD/LADucation.html where several examples and detailed instructions can be found.

Several examples of in-class applications will be demonstrated.
(Partially supported by projects ECP 2006 EDU 410016 Intergeo and MTM2008-04699-C03-03 Algoritmos en Geometría Algebraica de Curvas y Superficies)

Name	**Matija Lokar**
Affiliation	UL FMF, Slovenia
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Title	**Reuse of teaching materials**

Abstract

When we are working with e-teaching materials we much too often find that the authors of such materials, meant for the use of teachers in the teaching process, do not use the opportunities offered by the new technologies. All too often the materials are a monolithic block (or at least their main part is), constructed the way an ordinary book or workbook would be. This demands that the teacher take them as a whole, precisely in the order they were written in. Is that really necessary? Do all teachers need the same form of resources, do they want to use them in the same order, and do they want their students to see the same examples, do the same exercises? Why not use the possibilities that new technologies offer and at the very least give teachers the chance to adapt the materials to their own and their students' needs.

In the project Active Maths (http://am.fmf.uni-lj.si) we were basically concerned with making resources that can be changed and combined. Important part of our resources were tests and quizes, developed using different computer algebra systems and dynamic geometry systems. We also used system STACK, developed by Chris Sangwin from University of Birmingham. This project was a practical manifestation of the knowledge that was gained over the years of making and using e-teaching materials, namely that teachers want materials that can easily be changed and reused for their own specific needs. Different methods of use will be shown; from using simple links to the existing materials and direct copying of resources, to ways of changing, combining and using new resources.

Name	**Wolfgang Schreiner**
Affiliation	Austria Research Institute for Symbolic Computation (RISC), Austria
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Title	**On Proving Assistants in the Classroom (and Elsewhere)**

Abstract

We describe our motivation for and experience with the use of the interactive proving assistant "RISC ProofNavigator" in courses on "Formal Methods" for master students of computer science and mathematics. The tool can be used to quickly produce (by a combination of manual user interactions, automatic applications of logical inference rules, and automatic simplifications/decisions provided by an external satisfiability solver) formal proofs as they typically arise in the verification of computer programs. In the same way the software can be applied in other scenarios where the goal is (on the basis of the language of predicate logic) education in formal modeling and reasoning, which are a human key qualification in many modern professions.

This is a submission for Session ConvMathAssist.

Name	**Alla Stolyarevska**
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Title	**Iteration and recursion in dynamical programming**

Abstract

Nowadays the importance of applications of optimization methods in different areas of science can scarcely be overestimated. The knowledge and correct use of optimization methods give an invaluable advantage for higher school graduates while obtaining a job. Studying of methods of optimization is the integral component of the program of preparation of specialists in the area of computer science. Usually the course of methods of optimization gives consideration to the economic aspect (linear programming, the decision of transportation problem as a special case for linear programming problems), and the mathematical aspect (nonlinear programming, and dynamic programming). The experience of teaching of optimization methods for the students of the Eastern-Ukrainian Branch of the International Solomon University is generalized in the current work. The curriculum, besides the economical examples, gives more extensive materials concerning use of methods of dynamic programming for the decision of a different class of problems. Dynamic programming isn't hard, but a lot of script programmers don't know about this powerful technique. The special attention is given to interpenetration of iteration, recursion, and memorization at the decision of optimization problems by methods of dynamic programming. During studying students use the packages Excel, Maple, and the programming language C ++.

Name	**Walther Neuper**
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Title	**Common grounds for modeling mathematics in educational software**

Abstract

There is an abundant variety of mathematics assistants (MMs), successfully used in education. The MMs reflect the respective representations of mathematics objects in the various domains: geometry, algebra, numerical analysis and simulation, graph theory etc. And the MMs reflect various aspects and views of education.

Since, in principle, there is only one mathematics (with unified foundations commonly accepted today), and since all the variety of MMs should reflect that foundations, this talk asks the questions: (1) What are the common grounds for existing MMs ? (2) What are the principles the development of MMs might converge to ?

(1) Common grounds: Here a "step" is suggested as the minimal unit on common grounds; a step operates on an object (i.e. on an algebraic object like a term, an equation, a function, or on a geometric object, or on a graph like a dag, etc) within a logic and a context. The step is related to at least one theorem and results in a transformed object and in an updated context.
The talk will discuss how a step relates to rigorous foundations in logics, as well as to MMs in algebra and geometry (omitting graph theory et.al).

(2) Some principles for convergence: Since MMs represent formal mathematics, MMs might explicitly implement principles of computer mathematics. Since MMs "are (models of) mathematics", MMs might implement math knowledge in a human readable format (e.g. Coq, Isabelle, Mizar). Since formal logic is the basis of MMs but hard to learn, MMs might filter off for naive users. Since learners want to proceed on their own pace, MMs might uncover logical details on demand – providing continuous support from the introduction of variables up to academics.

The author looks forward to relate these suggestions to the variety of opinions presented in the working group "Convergence on Mathematics Assistants".

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Title	**Educational Meanings of Printed Materials with Three Dimensional Figures --- With the Use of KETpic to Insert Figures into LaTeX Documents ---**

Abstract

The authors have been developing KETpic as a bundle of macro packages for Computer Algebra Systems(CASs), such as Maple, Mathematica, Scilab and Maxima, to draw fine LaTeX-figures. KETpic enables us to provide teaching materials with figures which are effective for mathematical education: (1) Figures are drawn mainly with monochrome lines, therefore they are suitable for making explicit teaching materials which are distributed as handouts in mathematics class, (2) the plottings are accurate due to CAS, and enable students to deduce mathematical laws and (3) by using various functions in KETpic, we can easily generate rich variations of two or three dimensional figures.

Through the daily use of KETpic in making materials, we have realized that printed materials with proper figures are good assistants for students to get a clear understanding of mathematical concepts. Last year we carried out a questionnaire survey about teaching materials with figures directed at mathematics teachers at

56 national colleges of technology and at 17 universities in Japan. It was found that they found it hard to insert three dimensional figures into their materials when they wanted to do it.

In this presentation, we give an example of materials which include three dimensional figures drawn with KETpic. The aim of the materials is to make our students understand the concept of total differential of a function with two variables. We consider the surface S given by z=f(x,y) and the tangent plane PL to S at a point Q(a,b,f(a,b)). To explain the partial derivative f_x(a,b), we show the cross sections obtained by cutting S and PL with the plane y=b. Moreover, we can draw a magnified view of suitable figures near the point Q. We can illustrate clearly the total differential of f(x,y) using these materials.

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Title	**Enhanced Personalized Learning Support of Computer Algebra Systems**

Abstract

Computer mathematic tools such as computer algebra systems and computer theorem provers have supported mathematicians, scientists and engineers by automating tasks for decades. Computer tools for numeric and symbolic mathematics has also gained increasing interest in different educational settings. Application scenarios in secondary education focuses mainly on the support of solving physical and engineering problems and e-assessment scenarios. Exploring and applying mathematic rules and theorems is a new and promising application scenario which raises novel issues in the education community. Although such application scenarios may support modern learning and teaching approaches, existing tools have not sufficiently used so far the great potential of computer mathematic tools the could provide. Focusing on computer algebra systems, they should not only track, assess and support students' single steps in solving mathematical problems, they could also identify which mathematical rules and theorems have been applied as well as which of the concepts can be successfully solved and which are challenging for the students. This features may be used the basis to build fine-grained user profiles about students' skills and link them to the mathematic concept space which can be used for personalized learning and assessment activities. In this talk (paper), we will discuss our approach and first findings in this idea which is built on the ISAC computer algebra system and the Knowledge Space Theory modeling user profiles.

Name	**Helga Jungwirth**
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Title	**Through different eyes: a sociological approach to explanations in CAS-based mathematics classrooms**

Abstract

Studies about CAS in mathematics teaching present, and evaluate innovative learning environments, investigate effects on students´ development of mathematical concepts and procedures, shed light on the process of instrumentation, or analyze teachers´ didactical approaches to CAS-supported teaching. Analyses, however, that address CAS-based classrooms under the aspect of a subject-related, social practice co-constructed by the teacher and the students have not been conducted frequently yet.
This view was taken up within a project of qualitative research supported by the Austrian Ministeries of Education, and Science. Data consisted of lessons given by teachers who were experienced in computer integration and did their „business as usual" in teaching common mathematical topics of the curriculum. In my contribution I will focus on explanations for activities involving a use of the program. To foster arguing is a core concern of mathematics teaching, and CAS-based settings can promote that aim in a particular way. On the one hand, my findings show that teachers and students establish the common type of arguing. On the other hand, however, findings give evidence that explanations tend to be abbreviated, or, moreover, are established in a further way that does no longer follow common reasoning. I will give examples for that new

type. It may be puzzling at a first glance but it becomes plausible if all findings are taken into account. Accordingly, teachers and students have co-constructed CAS-based mathematics teaching as a „technologically shaped practice" having rules that differ from those well-known from mathematics teaching without any use of computers. The emerging divergence in explanation raises two issues for discussion that will be addressed in my contribution. First, long-term consequences for a promotion of genuin mathematical reasoning need attention, and secondly, possible impacts on students´ struggles with instrumentation have to be inspected.

Name	**Christian Hirsch**
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Title	**Innovation in Design, Access to, and Use of Software Tools for High School Mathematics**

Abstract

Since 1992, the Core-Plus Mathematics Project, with funding from the National Science Foundation, has been engaged in research and development of curriculum materials that interpret and implement professional recommendations for curriculum, instruction, and assessment in high school mathematics. Due to concerns for access and equity, the 1st edition materials were based on a modest technology assumption—students had access to graphing calculators for investigations and problem solving both in and outside of school. In current work on the 2nd edition, the contextual and mathematical problems that the curriculum is organized around and the learning expectations for students are such that it was desirable to augment graphing calculator use with computer tools. To meet this challenge and maintain access and equity, the project systematically explored the development of Java-based software that evolved into CPMP-Tools—a suite of general purpose and custom software tools, integrated with development of the curriculum materials.

The software suite includes tools for algebra (including a CAS), geometry (including a DGS), data analysis, and discrete mathematics (especially tools for constructing, manipulating, and analyzing vertex-edge graphs). The tools and their functionality are organized by Course needs to focus on the intended mathematics and to reduce the steepness of the learning curve. The tools are connected and share similar menu screens and user interfaces, thereby promoting integrated use and learning transfer from one tool to another. The tools are built using Java WebStart, which permits safe, easy, reliable distribution of software and updates across the Internet and on different types of computers. As public license software, CPMP-Tools is free to be used and further developed by others.

This session provides an overview demonstration of CPMP-Tools, discussion of design features and development decisions, and examples of how the software is being used to support mathematical explorations and problem solving.

Name	**Alexandru Horváth**
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Title	**From Discovery to Proof - a New Approach to an Old Theorem in Plane Geometry**

Abstract

Three points in the Euclidean plane in general position single out a fourth point in many ways -- these are in a sense centers of the triangle defined by the three points (see the "Encyclopedia of Triangle Centers" for more then 3000 such points). Let us now consider four points in general position, and ask the question if the quadrangle do has a "center", i.e. a point which is singled out by these points but not by any fewer then

four?
We propose a definition for such a point using Feuerbach's circles. Along the way we rediscover an old theorem of Brianchon-Poncelet (1820), and we find several interesting geometric properties, perhaps some out of them are new.
The main feature of our approach is the using of computer algebra and geometry: both the discovery and the proof is made by computer. The discovery were supported using Cabri Geometry II, while the proofs are given using the theory of Gröbner bases, which can be computed with the Buchberger's algorithm.

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Title	**In search of a locus using PC and DGS**

Abstract

We start by constructing a simple locus. By posing a well chosen 'Why if…' question a whole new arena for searching for new and more complex loci is uncovered. In this situation we try to follow up Polya's statement: Certainly, let us learn proving, but also let us learn guessing. This is the time and place for experimentation. Construction, testing of similar elements and looking for invariants are typical activities when trying to formulate a conjecture. A computer with a DGS (Dynamic Geometry System) is a powerful tool in investigations of this kind, but the work also demands mathematical knowledge and creative thinking. This tool can easily and quickly generate a large number of geometrical elements so that guessing and inductive thinking may help in the search. In addition, the students will have to use mathematical problem solving strategies which involve them in what we generally call mathematical thinking. Consequently, the work with loci represents an exciting arena for learning and practicing of mathematics.

Also in the next phase, where we try to verify our conjecture, the DGS will have a prominent place. Two of the commands in GeoGebra that are useful in this part of our investigation are 'Locus' and 'Relation between two objects'. You will find similar commands in other DGSs. The activities described so far are without doubt feasible in lower secondary school.

Induction of the kind described above, can make us pretty certain that we have found the correct relationship, but it does not give us a formal proof. A formal proof is only possible when the students have become familiar with analytic plane geometry, usually in upper secondary school. We will conclude by showing how this can be carried out.

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Title	**Integrating DG and CAS abilities under a common framework - Session DEIM**

Abstract

Recent versions of DG software share a common trend: since the users claim for more sophisticated calculus abilities, DG developers add new resources to their software. On the other side, some CAS begin to offer some limited dynamic functionalities. Nevertheless, the current practice in mathematical education usually involves using both systems. In this talk, we describe two cases of successfully integration of DG resources with CAS abilities through SAGE, a free open source system developed as a viable alternative to expensive and opaque common systems such as Mathematica, Maple, ... Furthermore, the notebook feature of SAGE allows users experiment with the material over the internet, without installing anything on the personal

machine.

The first illustration deals with learning Lagrange multipliers in functions of two variables. A Cinderella applet showing the possible conditioned extrema as the points where the gradients are parallel is integrated into a SAGE worksheet. Additionally, another applet is used, where the student can approximately test the candidate points. Last, basic commands of SAGE confirm the previous results.

The second example shows the synergies between DG and CAS when finding a geometric locus. Using GeoGebra as an applet inside SAGE, the problem of determining the positions for a triangle in order to satisfy a condition involving its bisectors. The problem, although easily posed is not trivial, and, as far as we know, not soluble inside the system. The user can only guess a rough solution. The symbolic solution is then provided via a simple Groebner approach by means of the SAGE's Singular interface.

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Title	**Answers Offered by Computer Algebra Systems to Expression Transformation Exercises**

Abstract

Mathematics lessons, textbooks and tests frequently require transformation of expressions (e.g., by simplifying, expanding or factoring). The requirement may or may not be mentioned explicitly in the text. At the same time, computer algebra systems (CASs) have a wide range of commands for expression transformation. This raises questions about the use of the CASs in transformation exercises in schools. The use of the CASs could lead to a reappraisal of the time and effort dedicated in the curriculum to the training of manual transformation skills. Any informed decisions in this field would require an overview of the answers offered by the CASs.

The paper investigates the handling of expression transformation exercises from school mathematics by the CASs. The classification of expressions developed for this research covers the majority of school lesson topics. Dozens of problems were solved with nine different CASs (Derive, Maple, Mathcad, Mathematica, Maxima, MuPAD, TI-92 Plus, TI-nspire and WIRIS), using both the basic commands (simplify, expand, factor) and some advanced ones. The aim was not to analyze the performance of particular CASs but to discover general trends.

The research indicates that the systems are mostly reliable and produce reasonable answers. Some distinctive properties of the answers are easily explained by built-in standards and notions, which may differ from the assumptions used in schools. The performance of all systems was relatively uniform and they gave virtually the same results to all test expressions. However, there were differences in the definitions of certain terms (like simplify) and in the format of answers. The paper also discusses questions related to number domains and special cases, as well as the usefulness of the answers, which differ from the expected result, and their relevance for the manipulation of expressions in CASs.

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Title	**JSXGraph - Dynamic Geometry with JavaScript**

Abstract

JSXGraph (http://jsxgraph.org) is a library for interactive geometry, function plotting, graphs, and data visualization in a web browser. It is implemented completely in JavaScript and uses SVG, VML or Silverlight, no further plug-ins are required. JSXGraph is easy to embed and has a small footprint: only about 55 KB if

embedded in a web page. JSXGraph runs with all major web browsers.
JavaScript still has the reputation of being slow. But meanwhile, the execution speed of JavaScript in the web browsers has been improved by some orders of magnitude and speed is still one of the big issues in the competition between the web browsers. A further advantage over Java programs is the much reduced download time and the zero initialization time.
A further reason for the use of JavaScript is that the future support of Java plug-ins by the web browsers - especially on small devices - seems
to be more than uncertain.

JSXGraph contains functions for
- Euclidean Geometry: Points, lines, circles, intersections, perpendicular lines, angles,
- curve plotting: function graphs, parametric curves, polar curves, data plots, tangents, normals,
- turtle graphics,
- charts,
- sliders, gliders.

Since JSXGraph itself is implemented in JavaScript the canonical scripting language for JSXGraph is JavaScript. This eases the integration of a construction in a web page and the connection to server based software like R (Statistics) or Cocoa (Computer Algebra).
JSXGraph supports the GEONExT and the Intergeo file format. Further file formats are under development.

Session FTIG

Name	**Kaja Maricic**
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Title	**The metodical computer visualization of basic functions and their inverse functions**

Abstract

Although "Obvious is the most dangerous word in mathematics" (E.T. Bell), the visualization of abstract mathematical notions is inevitable in teaching in order to help students to acquire knowledge more easily. According to the present curriculum in Serbia, at the beginning of the last grade in secondary school (before introducing the notions of a limit, a derivative etc.), it is necessary to do a methodical review of basic functions (the functions which are familiar to students after the units completed in previous grades) – their graphs and their properties which could be red from the graphs. It is also necessary to clarify notion of the inverse function. An example of computer visualization of both will be presented in this paper.

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Title	**Issue of financial literacy**

Abstract

In the late 2007, the government of the Czech Republic approved document "The system of financial literacy building-up at primary and secondary schools" reflecting the recommendations of European Commission for the field of the policy of financial services.
At the same time, the government asked representatives of the universities preparing the teachers of mathematics at primary and secondary schools for the incorporation of the issue of financial literacy into the contents of the relevant university curricula. Pedagogical Faculty of the University of South Bohemia in České Budějovice reacted to this challenge by the establishment of subject An Introduction to Finance incorporated into all curricula of the University of South Bohemia as optional subject.
For most inhabitants, i.e. also students, it is not easy to have a good grasp of basic terms in the field of

finance and of their interrelations, in particular due to the fact that the offer for financial products keeps changing and its statement is frequently unclear, insincere and confusing.
The authors of the article created a multimedia aid that would be helpful to students for their better grasp of the issue of money and prices and for their responsible administration of family budget as only a well-educated and well-informed consumer of financial products is a full-fledged player in financial market. To facilitate the understanding these concepts the multimedia aid uses tools of the computer algebra system Maple.
The article familiarises readers with this multimedia aid along with the examples of its utilisation and the results achieved during teaching of subject An Introduction to Finance.

Name	**Rein Prank**
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Title	**White-Box/Black-Box Principle in Expression Manipulation: How Much Can Be Automated?**

Abstract

The paper analyzes known theoretical results concerning the possibility of construction of necessary computational kernels for learning environments in step-by-step expression manipulation.
We consider separately:
- input-based interfaces where the student enters at each step the result of conversion and the program checks its correctness (example: Aplusix),
- rule-based interfaces where the student marks a subexpression for the step and selects from the menu a conversion rule, which will be applied by the program (example: MathXpert, but also computer algebra systems).
Input-based work can be used for automated checking of manual calculations at White-Box stage and rule-based work is the same as the Black-Box stage of Principle.
The key issue in building input-based environments is the solvability of equivalence of expressions in 'area X', because at each step the program should check equivalence of the entered expression with the previous one. We draw a line between possible and impossible, using positive and negative results of D. Richardson and Y. Matiyasevich about expressions containing different sets of elementary functions. We also discuss the approach, which is based on evaluation of expressions by random values of variables, and recent examples of non-equal numerical expressions with extremely small differences. In addition, we consider logical expressions in propositional and predicate calculus.
For the rule-based approach, it is desirable to have a complete set of rules where for each pair of equivalent expressions, one expression can be transformed into another using the rules. In some cases such set of rules does not exist. Our discussion here focuses on school algebra, the positive and negative results with Tarski's School Identities, propositional and predicate logic.

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Title	**Some New Regular Toroids with Hexagonal Faces**

Abstract

We present examples of polyhedra homeomorphic to a torus such that all of their faces are planar polygons with the same number of sides and all of their vertices are incident to the same number of edges. Neither the faces nor the polyhedra are self-intersecting. We call them regular toroids. Because of their combinatorial regularity, as a kind of symmetry, these structures may be attractive to people who find beauty in geometric patterns
We show that for all F, 7≤ F ≤ 12, there exists a regular toroid whose faces are simple hexagons. For all combinatorial types of these we present an example such that two distinct face have no point in common,

apart from a vertex or edge they possibly share. We give the combinatorial structures of these polyhedra together with the coordinates of their vertices.

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Title	**Dynamic Geometry vs Game Physics software - Session FTIG**

Abstract

Since its origins, Dynamic Geometry has attracted a lot of attention from math teachers, but it does not really seem to engage the students. Very different is the case of a new class of software that is spreading rapidly among youngsters while hardly getting any attention from math instructors. This kind of software, generally referred to as "Game Physics", offers an environment where Newtonian laws are simulated allowing the user to play with a virtual microworld of simple objects. The idea is not new to DG tools which are frequently used to simulate physical situations. Indeed, some DG programs offer virtual laboratories where a user can make experiments in a physical setting.

In this talk we use a recent piece of physics software, Phun, to discuss the applicability of game physics software for the learning of some geometric aspects, such as loci and mathematical machines. We test Phun with the classical four bar linkage in order to describe the Watt curve and show its ease of use when reproducing mathematical mechanisms. Pros and cons of the physics approach will be considered, and a proposal for using it as an experimental step previous to creating a DG construction is given.

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Title	**Establishing the Geogebra Institute of Norway**

Abstract

GeoGebra (http://ww.geogebra.org) is open source dynamic mathematics software with rapidly growing worldwide popularity, especially in Europe and North America. The basic idea of its development is to create a dynamic software that incorporates geometry, algebra, and calculus, which other packages treat separately, into a single easy-to-use package. A large international user and developer community have grown around GeoGebra.

To support teachers, develop GeoGebra in general and coordinate research in relation to GeoGebra, an umbrella organisation, the International GeoGebra Institute (IGI), was established in the past year. The first local GeoGebra Institute was established in Norway with the support of Norwegian Centre for Mathematics Education and the Norwegian University of Science and Technology. The poster explains the development of the GeoGebra Institute of Norway (NGI) and its aims for supporting teaching and learning of mathematics in Norway and contribute to the work of IGI.

The poster outlines the importance of research on supporting teachers to be able to integrate technology into their teaching. Research suggests that, for the majority of teachers, solely providing technology is insufficient for the successful integration of technology into their teaching. The poster outlines the mission and current and further activities of the NGI in the following order:

Training and Support: Starting with the Norwegian Centre for Mathematics Education's network of teachers spread all over the country, we have build a national network of certified GeoGebra trainers.

Development and Sharing: GeoGebra Institute of Norway will take special responsibility for developing of materials well adapted to the national curricula in Norway.

Research and Collaboration: The Norwegian Centre for Mathematics Education coordinates a national network of mathematics teacher trainers and researchers. The Institute will use this network for collaboration with the other universities and university colleges in Norway.

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Title	**On the visualization of function concept by using CAS**

Abstract

We investigate the application of Computer Algebra System (CAS) in teaching and learning functions in order to avoid cognitive difficulties and conflicts in transition to advanced mathematical thinking.
The programme packages *GeoGebra* and *Scientific Workplace* will be used for visualization of the contents. In particular, the graphs of logarithmic, exponential and trigonometric functions will be considered.
The advantages and disadvantages of the applied procedures will be emphasized and analyzed. Moreover, the opinions of high school students and teachers about the used procedures will be presented.

The part of questionnaire is:

1. Determine, with and without computer the function f such that the function g is its derivative and the function g is given by:

   a) $g(x)=\frac{1}{x^2}$, b) $g(x)=\frac{1}{x}$, c) $g(x)=\frac{1}{\sqrt{x}}$, d) $g(x)=\frac{1}{x^a}$, $a \in R$,

   and explain the answers.
2. Determine, with and without the computer, the functions

$$\int g(x)dx,$$

   where the function g is given in example 1., and explain the answers.

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Title	**On the notions of function**

Abstract

We present the introduction of the notions of function, at the begging of the fourth grade of Grammar school by using program package GeoGebra.
We investigate the student's knowledge of definition and notions of function at the end of Novi Sad Grammar school.
The emphasis of the work is on testing three different classes in fourth grade of secondary school.
The questionnaire, considered in paper of David Tall ([2]), is given to our students and the obtain results are analyzed.
Two groups, of considered three groups, were taught by using program package GeoGebra, and the third one was taught without computer.

Name	**Manfred J. Bauch**
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Title	**Thoughts On The Future of the Classical Worksheet**

Abstract

A look at German classrooms shows that the traditional paper-based worksheet has not vanished. Quite the contrary, its deployment has increased. On the other hand, the notion of a worksheet also appears in the context of dynamic geometry and computer algebra systems.

Between these two extremes, one can find a large variety of modifications that give rise to the question about the future of worksheets: Will electronic and paper-based version coexist, will they merge or will one form or the other vanish?

We present several examples and discuss elementary characteristics of a „worksheet", in particular their dependence on certain media.

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Title	**Explorations in Elementary Mathematical Modeling**

Abstract

In this paper we will share the methodology and pedagogy of Elementary Mathematical Modeling as a one-semester course in the liberal arts core. The main mathematical tools in this course are difference equations and matrix algebra. We also integrate computer technology and cooperative learning into this inquiry-based learning course where students work in small groups on carefully designed activities and utilize available software to support problem solving and understanding of real life situations. We emphasize the use of graphical and numerical techniques, rather than theoretical techniques, to investigate and analyze the behavior of the solutions of the difference equations.
As an illustration of our approach, we will show a nontraditional and efficient way of introducing models from finance and economics. The students use first-order non-homogenous difference equations in the form $y(n+1) = ay(n) + b$ to model annuities, sinking funds, mortgages and amortization. They use a computer algebra system, graphing calculator, or spreadsheet to iterate the difference equations to find the solutions and investigate the behavior of the graphs. The students also can find the solutions of such problems by utilizing the analytical solutions of the difference equations. We will also present an interesting model of supply and demand with a lag time, which is called the cobweb theorem in economics.

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Title	**Establishing a professional development network with an open-source dynamic mathematics software - GeoGebra**

Abstract

In this talk, we will outline the initial stages of an NCETM (National Centre for Excellence in the Teaching of Mathematics) funded project that aims to establish a professional development network with an open-source mathematical software – GeoGebra – in England. During the past years a large international user and developer community has formed around various GeoGebra-related activities. Most teachers who are currently using GeoGebra have not received professional training in the implementation of the software in their teaching practices, but have begun using it due to their enthusiasm or encouragement by their colleagues. However, research suggests that, for the majority of teachers, solely providing technology is insufficient for the successful integration of technology into their teaching. It has been suggested that adequate training and collegial support boost teachers' willingness to integrate technology into their teaching and to develop successful technology-assisted teaching practices. Thus, the project, involving nine experienced English teachers, aspires to review, to modify, and to implement already existing teaching materials for the use in English classrooms and to develop ways of providing professional development and support for teachers in England. In this way, we hope to nurture a community of teachers and researchers in England who are interested in developing and using open-source technology in schools and in teacher education.

Name	**Ka-Lok Wong**
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Title	**Development of Technological Pedagogical Content Knowledge: Mathematics Teachers Designing a New Kind of Dynamic Geometry Manipulative Tasks**

Abstract

Most mathematics teachers nowadays are familiar with dynamic geometry (DG). Dynamic geometry provides a supportive environment for exploratory activities focusing on students' reasoning in terms of conjecturing and generalization. However, while the geometric properties to be learnt in mathematics classrooms are usually the theorems already well illustrated by textbooks, mathematics teachers often find little room for students to explore. As a result, classroom use of DG often takes the form of demonstration of invariant geometric properties in a dynamically changeable geometric figure. In view of the gap between such limited DG usage on the one hand and students' paper-and-pen exercise and geometry questions in assessment on the other, the author and others have been trying out a new kind of DG manipulative tasks which assume no operational skills with any particular DG software on the part of teachers and students but only mouse clicks and drags. Pedagogy is not a question of what is available to the teachers but how the teachers are going to use it. Following the conceptual framework of "technological pedagogical content knowledge (TPCK)" suggested in recent years by Matthew Koehler and Punya Mishra, the author argues for the usefulness of engaging a group of teachers in the process of designing the tasks in shaping their understanding about the tasks as well as their conception about their pedagogical use. Examples of teachers' design of such DG manipulative tasks on elementary geometric concepts are presented to illustrate how this process facilitates an integration of all the three aspects: technological, pedagogical and mathematical concepts in teaching geometry, and how, once again, a new technological possibility opens an opportunity for professional teachers to rethink even very simple concepts.

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Title	**A didactical aspect on teaching mathematics using dynamic geometry software**

Abstract

The article summarizes some preferences of dynamic geometry software (DG) and it points on connection among theoretical mathematical concepts and the concepts dynamically symbolised by the software. It also didactically analyses some traps which may prevent student's theoretical development of mathematical concept. As it is specific for each model or didactical tool dynamic geometry has specific (software) solutions, which differ from usual symbolic representations or procedures using pencil and paper. These differences can be the obstacles on the way of meaningful software usage and students' conception of mathematical notions. The obstacles can be exceeded and removed simultaneously, if teachers are aware of them. The article covers some rarely recorded obstacles meeting while students learn geometry concepts with dynamic software like understanding intersection point, segment, dependant and independant properties.

Name	**Mette Andresen , Morten Misfeldt**
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Title	**Essentials of teacher training sessions with Geogebra**

Abstract

Formal requests were recently introduced for integration of ICT in secondary school mathematics. As the main issue, students must develop competence to decide when and how it is appropriate to use available ICT tools and to use them. These new requests put demands on those teachers who have not developed corresponding competencies themselves.
Danish GeoGebra Institute designs training sessions for teachers, which aim to initiate or support professional development to meet such needs. The design of these sessions does not see mastering the software GeoGebra as an end but as a means for the teacher to reach his teaching goals. Although we see good reasons to choose GeoGebra, other software might serve the same purposes.
The paper presents our model for the design of teacher training sessions aiming at ICT – integration. The model's designs are centred on four essentials:
- Tool: Mathematical content, instrumentation (Task – student - computer)
- Medium: In and out of the computer, express meaning, read and write mathematics, mathematical work as manipulation of semiotic representations
- Vehicle for learning: Flexibility of concepts, mathematical reflections, potentials for change of focus on concepts, processes and relations rather than on technical details. Possibility of creating mathematical micro-worlds and games.
- Change agent (LCM):
o Rethinking of teaching mathematics: teamwork, interdisciplinary projects, new roles for the teacher
o New perspectives: modelling aspects, applications, authentic models
o New content: Numerical methods, fractals etc.

First try out of our model, in a recent project in a small school with ten teachers, is reported in the paper. The project aimed to create a platform of shared knowledge about mathematical activities and issues to practice in the every day teaching, including the use of ICT.

Name	**Rita Nagy-Kondor**
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Title	**Descriptive geometry and Dynamic Geometry Systems**

Abstract

Dynamic geometry systems offer new opportunities for the teaching of descriptive geometry. These systems make possible to create drawings quickly, accurately, and flexibly. According to our results so far we can say that DGS and the use of interactive worksheets help the teaching of the descriptive geometry and it is also a useful help to improve the students outlook in geometry.

Name	**Mihály Klincsik**
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Title	**Effects of CAS on understanding probabilistic concepts**

Abstract

We involve the Maple Computer Algebra System (CAS) in our probability and statistics courses as an interactive tool and investigate the question: "Can the Maple provoke accelerative effects on the learning process?" The main features of the CAS are certainly applied to achieve the desired results such as

• Calculating probabilities with large accuracy in which are occurring big numbers
• Creating small programs to compose by this way various sets, lists and sequences with great number of elements so we can enumerate the elements of them
• Manipulating symbolically with different random variables and asking from the Maple CAS to answer for specific features of them
• Visualizing graphs of the well-known distributions and making animations with them
• Experimenting with the system to obtain some new results.

We combine the procedures of the Maple from the "combinat" and "Statistics" packages so that the students can get closer to understand the mathematical concepts of the randomness. With these tools the students can gather some experiences in the teaching environment about the random phenomenon which are occurring in complex engineering situations? So the students can distinguish different types of probabilistic and statistical models and they can involve the CAS objects besides of the mathematical objects into the problem solving process. We are handling classical problems in our lecture from the probability theory and investigating the properties of random variables by using the Maple CAS capabilities.
By this way we can achieve more or fewer outcomes in moving the student's knowledge from the numerical skills toward to the conceptual understanding of the models because of the CAS reduce the complexity of the problem and deliver the difficult and accurate numerical, symbolical calculations and the visualizations to us. In our presentation we give examples on practical sessions to demonstrate how can we create sets on the basis of given combinatorial assumptions and work with random variables.

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Title	**Numerical Integration with GeoGebra in High School**

Abstract

We have created a set of GeoGebra tools and related teaching materials for numerical integration, which we use in teaching numerical mathematics at high school level, and also at undergraduate level at the University of Novi Sad. Primitive quadrature rules, trapezoidal rule and Simpson's integration are covered. A new, interesting approach to implementation of numerical integration is taken, using new features of GeoGebra 3.2. Our experience from teaching numerical integration using GeoGebra will presented.

Name	**Zoltán Kovács**

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Title	**Making a difference**

Abstract

The use of difference and differential equations in the modeling is a topic usually studied by advanced students in mathematics. However difference and differential equations appears in the school curriculum in many direct or hidden way. Difference equations first enter in the curriculum when studying arithmetic and geometric sequences. Moreover Newtonian mechanics provides many examples for differential equations and numeric solution leads to difference equations which can be treated easily with Computer Algebra Systems. In the talk I show some detailed examples with the help of the open source CAS Maxima. My hypothesis is that numerical methods supported by technology serves a tool which helps the early introduction of modeling concepts.

Name	**András Ringler**
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Title	**How to construct a regular pentagon from a square?**

Abstract

The lecturer will show, how to get a regular pentagon from a square; how to construct distances, being in golden ratio.
If an ABCD square, with a side length a is given (a º AB = BC = CD =DA), then
around the middle point of the AB side one has to draw a circle going through the points C and D. This circle and the straight line containing the points A and B cross each other in the points E and F. The points E, F and D being on the circle form a Thales-triangle.
The height of the EFD triangle, belonging to the EF hypotenuse, the AD º a distance is the geometrical mean of the EA º b and the AF º a+b distances: a2 = b× (a+b).
The a2 = b× (a+b) equation is equivalent with the b : a = a : (a+b) equation, which
represents the golden ratio, known from the secret symbol of the Pythagoreans. At the beginning of the construction, the b distance is unknown, then by the drawing the circle, the distance x = b is one solution of a so called hyperbolic symptoma (equation), given in the form a2 = x× (a+x) = a× x+x2.
It is interesting that the distances a and x = b are in golden ratio, so with these two distances the "pentagramma mirificum", the secret symbol of the Pythagoreans can be constructed, step by step, with a compass and a ruler.
If in the a2 = a× x+x2 equation the a distance is the unknown (a = y) and the x = b
distance is the known one, then one has to solve the y2 = y× b+b2 Þ b× y - y2 = - b2
elliptical symptoma (equation).
To solve (such kind or other type of) quadratic equations with a compass and a ruler is very interesting to students and to professors too. The link
www.mozaik.info.hu/homepage/mozaportal/matematika.php shows surprising geometric possibilities at 4 different symptomas (equations). The animation can be seen on line, but there is a possibility to make a free copy about the program.
Till to day the number of downloading is over 8000.

Name	**Josef Böhm, Eno Tonisson**
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Title	**Do we need a CAS-dictionary?**

Abstract

Many CAS users - especially those who use a CAS as an educational tool - are facing exciting times. There are several CASs available and the situation may change quite rapidly. The new CASs could appear and existing ones could fade away. For example, DERIVE is off the market and there are a couple of competitors for taking the leadership as a classroom tool.

Teachers have the choice between many products and they might try various products before deciding what to recommend to the school authorities, the parents, the colleagues and last but not least to the students.

We wonder if something like a 'vocabulary' or 'dictionary' for the most important activities supported by a CAS (simplification, solving of equations and inequalities, drawing graphs of the functions etc.) might support the teachers and students as well when changing from one system to the other. The commands are very similar but often tiny differences in syntax and output of the results can cause problems.

The possible product (vocabulary, dictionary, ...) could form a base for people who would like to create a new "pedagogical CAS". Also, it could be useful for the researchers who compare different CASs.

The product should be web-based, user-friendly and open for supplements. In a sense the multilingual dictionaries could be taken as the models. In our case, the commands of certain CAS form the "language" (or "dialect"). On the other hand there are several special needs.

We try to show some possible technical solutions. However, our main goal is to discuss the reasonability of such a dictionary in the conference.

Name	**André Heck, Ton Ellermeijer**
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Title	**Mathematics Assistants: Meeting the Needs of Secondary School Physics Education**

Abstract

Coach is an activity-based, open computer environment for learning and doing mathematics, science, and technology in an inquiry approach, developed in the last twenty-five years at the AMSTEL Institute of the University of Amsterdam. It offers a versatile set of integrated tools for data collection, data analysis, modeling and simulation, and for multimedia authoring of activities. We present the STOLE concept, which is an acronym for Scientific and Technical Open Learning Environment, underpinning the design and implementation of Coach. It is an example of how members from the physics education research community came to convergence on tools for doing investigative work and achieved integration of tools.
Special attention goes further to the mathematical requirements of the learning environment and to the computer support of various representations of one and the same phenomenon or scientific concept. We also discuss one of the most complicating factors in the implementation of an integrated learning environment for mathematics and science, namely that mathematical concepts are not always used the same in these fields. Differences between the use of variables, functions, and graphs in mathematics and physics are briefly discussed, and consequences for the design of a general-purpose learning environment are addressed.
We exemplify what has already been realized in the Coach environment with regard to mathematical assistance, how these tools have been applied in real education, and which extensions of mechanized mathematics assistance are in progress or still awaiting functional specification.

Name	**Makarius Wenzel**
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Title	**The Isabelle/Isar framework as a "logical operating system"**

Abstract

Isabelle was originally conceived as a "logical framework" by L. C. Paulson in 1989. The meta-logic of Isabelle/Pure implements a bare minimum of higher-order natural deduction that enables users to specify a variety of object-logics, by declaring connectives and rules. This facilitates experimentation with basic logical calculi, although a realistic working environment will demand development of specific theory libraries (with numerous definitions and proofs, and add-on tools implemented in SML).

On top of the Pure framework, the Isar layer was added around 1999 by the author, in order to enable "human-readable formal proofs" (similar to Mizar). Implementing structured proof processing is a very demanding task, but our systematic approach allowed to single out general principles of organizing formal reasoning beyond the primitive calculi. The main Isar concepts are that of a theory and proof context, with generic data declared by user code, where the framework manages correctness wrt. monotonic extension, and transfer from one context to another.

This basic Isar infrastructure has been refined and extended further in recent years. There is now a general framework for "local theory specifications", which supports a variety of module concepts on the one hand (e.g. Isabelle locales, type classes, class instantiation, theory interpretation), and combines them with derived specification mechanisms on the other hand (e.g. inductive predicates, recursive functions).

The user can implement derived Isar language elements specifically as "proof methods" or "theorem attributes", or as arbitrary "commands" of the toplevel language. Ultimately, these language elements perform transactions on a formal document model, which are managed by the system in a valued-oriented fashion. The recent addition of implicit parallelism (with automatic proof scheduling) illustrates particularly well how the initial "logical framework" is evolving into a "logical operating system". Thus it may serve as implementation platform for advanced mathematical assistants, beyond traditional interactive theorem proving.

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Title	**Design of M@th Desktop (MD): Considerations of software design and how to teach and learn with M@th Desktop**

Abstract

M@th Desktop is an e-learning software based on Mathematica for teaching and learning Mathematics. The user of MD are teachers and students at high schools, universities of applied science, undergraduate students of science and technology.

Design concepts of the learning environment MD
The basic elements of MD are palettes and notebooks. The palettes contain the „mathematics" necessary to deal with problems of a certain topic e.g. optimization. The accompanying notebook shows demonstrations, examples, exercises and a Test Your Knowledge section.
In the talk the role of different teaching styles with MD is discussed. The blended learning concept will take center stage.
The palette-notebook design of MD allows for a relatively simple handling of the courseware. Only a little knowledge of Mathematica is required. This is especially useful when teaching in big classes.

Customizability of MD
Each user, teacher and student as well, can design and develop own palettes. Students like to do it before test.

The teacher has MD's tools to his/her disposal to edit the existing notebooks or write his/her own content, prepare practice sheets, tests, etc.
In M@th Desktop there are units for polynomial functions, exp, log and trigonometric functions, data fitting, differentiation, integration, linear algebra and statistics.
The units cover the standard curriculum of EU and US high schools.

EU Projects
M@th Desktop has been adopted in 3 Comenius projects for high schools as teaching software. During the LTM Comenius 2.1-project in 2005-2008, coordinated by the University Graz, Austria, two further modules of MD were developed.
LTM homepage: http://ltm.uni-graz.at/
MD Homepage: http://www.deltasoft.at

Name	**Katarína Žilková**
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Title	**WebMatika.sk – School Mathematics in the environment of ICT**

Abstract (Poster)

Visualization, representation, interactivity and dynamism, modeling and simulation are among the most important factors that influence mathematics teaching and learning. DG systems allow us creating geometrical models that embody all these attributes.
The website www.webmatika.sk offers interactive animations together with the instructions on how to create them. Their main purpose is to provide visualizations of fundamental and derived geometrical concepts and explanation of various procedures. Offered animations are of different kinds – animations based on the platform of current DGS, animations based on the platform of professional graphical systems, animations combined with virtual video and animations combined with realistic video.
Geometry assignments form an important component of the website. They are based on DGS platform and provide an option to adjust the difficulty level of the task by tools selection and provide feedback to the solver if s/he successfully completes the task.
Webmatika contains products that draw on tools of DGS such as Cabri II Plus, Cabri 3D and Compass and Ruler (C.a.R.) and allow the user to explore geometric figures and their attributes as much by the vision as by virtual manipulation mediated by computer mouse and software tools.

Name	**Tatsuyoshi Hamada**
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Title	**The toy box of mathematics: KNOPPIX/Math**

Abstract

KNOPPIX/Math is a project to archive free mathematical software and documents and offer them on KNOPPIX, a bootable CD/DVD that contains a collection of GNU/Linux software.
The KNOPPIX project was started in Germany by Klaus Knopper. KNOPPIX can be used for Linux demos, educational presentations and system recovery.
KNOPPIX/Math provides a desktop for mathematics that can be set up easily and quickly. The KNOPPIX/Math project began in February 2003. The newest edition is KNOPPIX/Math/2009, which contains many open source dynamical geometry software: C.a.R., DrGeo, GeoGebra, GEONExT, KSEG, KidsCindy, Kig, PyGeo, and GeoProof. Once you run the live system, you can enjoy a wonderful world of dynamical geometry software without needing to make any installations yourself. KNOPPIX/Math also includes many computer algebra systems, documents, sample files and flash movies. As an experimental attempt, KNOPPIX/Math has full-text search capability for mathematical documents.

Name	**Predrag Janicic**
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Title	**Automated Geometry Theorem Proving: Readability vs. Efficiency**

Abstract

There are several very efficient methods for automated geometry theorem proving, including Buchberger's and Wu's methods.
However, these methods are primarily algebraic methods and do not produce traditional and readable geometry proofs. On the other hand, there are some proving methods (e.g., based on coherent logic) that can prove geometry theorems in traditional manner, but the power of such provers is much less. Typically, these two families of theorem provers have different scopes and languages. Similarities and differences between these two families will be discussed and illustrated by the geometry tool GCLC and its built-in theorem provers.

Name	**Jürgen Richter-Gebert**
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Title	**Visualization and Algorithms: Old teaching paradigms in new contexts**

Abstract

The tradition of teaching mathematics and/or computer science via hands-on computer exercises has passed through various crucial stages. This talk demonstrates how old paradigms from „fast visual feedback" languages like LOGO are revitalized in modern Dynamic Mathematics/Geometry software. The possibility to combine an intuitive, easy-to-learn, fast-prototyping programming environment with the interactive power of a dynamic geometry system opens a vast area of new possibilities as well for teaching as for learning both mathematics and computer science.

The talk presents several scenarios that take advantage of this combination of algorithms and visualization. The language CindyScript is designed to meet the above mentioned requirements. We will demonstrate how already a few lines of code lead to fascinating and far reaching mathematical experiments. The examples will not only be taken from various classical areas of mathematics like: geometry, calculus, statistics, fractals, dynamical systems. We will moreover also touch more exotic topics like mathematical theory of sounds, plant growth, population dynamics or experimental physics.

As a side effect we will demonstrate that combining algorithmic possibilities with well known approaches from dynamic geometry create very versatile systems that can be used for visualizations far beyond geometry that cover essentially all of mathematics and many neighboring fields. Many of the examples presented in the talk can be found on the portal www.mathe-vital.de or in the example gallery in www.cinderella.de.

Name	**Ulrich Kortenkamp, Christian Dohrmann**
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Title	**User Interface Design for Interactive Geometry Software**

Abstract

For the user interface of a dynamic geometry software system multifaceted questions concerning the (a)

usability, (b) functional aspects, and the (c) pedagogical benefit must be considered.
Based on these aspects this paper illustrates and categorises different approaches of various software systems to emphasise existing problems as
(1) The discrepancy between providing a wide functional range while maintaining usability,
(2) The discrepancy between specialised tools for DGS and commonly known tools in plain drawing software,
(3) The discrepancy between the use metaphors of traditional geometry tools like ruler and compass and their computer versions,
(4) The duality of geometric objects and geometric relations,
(5) The specification of order for objects, and the
(6) Object-Action vs. Action-Object distinction.
Furthermore, we discuss the use of new technologies like multitouch input devices. These are a possible approach to expand the user interface through gesture based activities that (1) support an intuitive way of interaction with the software itself (usability), and (2) implement a set of knowledge based gestures as a basis for the problem solving process (pedagogical benefit) within the software.
Following this analysis the paper provides first solutions and a concept of a DGS user interface that has been developed with usability aspects, functional aspects and pedagogical aspects in mind. These results have been developed in cooperation with human interaction design experts of the University of Design Schwäbisch Gmünd.

Name	**Ulrich Kortenkamp**
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Title	**Blended Experimentation with DGS**

Abstract

Experimentation is an important element of science teaching. Students should experience real-world phenomena, and support or disprove their hypotheses by setting up appropriate situations, conducting the experiments, gathering data and drawing conclusions. Unfortunately, many experiments are unsuitable for school teaching out of various reasons. They might be too dangerous, too expensive, too complex, too unreliable, or they take too much time to complete.
Using simulation software, many experiments can be replaced by safer, cheaper, easier, more reliable or sped-up virtual counterparts. The simulations can be done, for example, by showing videos of repeated runs of the real experiment, together with interaction facilities, or by mathematical simulations using numerical solutions of partial differential equations.
However, there is a pedagogical drawback of the simulation approach: The connection to the real world situation is lost. This might harm both the motivation of the students and their belief in the conclusions.
In this paper we describe the approach of blended experimentation, where simulations and the real world are connected via sensors and actors. The sensors are able to influence the simulation, while the actors can change the real world. We show first implementations and examples using Dynamic Geometry Software as simulation environments.

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Title	**Enhancing mathematical communication for virtual math teams**

Abstract

The Virtual Math Teams (VMT) Project has conducted research for the past six years on how to support

small teams of students around the world to collaborate online in discussions of stimulating mathematical topics. The project has developed an extensive web-based environment and conducted about 400 sessions of usage. Analysis of usage has resulted in over a hundred academic publications—the most important of which are collected in Studying Virtual Math Teams (Stahl, 2009)—and doctoral dissertations (see summaries in Çakır, Zemel, & Stahl, 2009; Sarmiento & Stahl, 2008).
The VMT environment currently includes a social-networking portal, a Java application that integrates synchronous text chat with a shared whiteboard, social awareness indicators, and an asynchronous community wiki (http://vmt.mathforum.org/VMTLobby/). The project plans to integrate the dynamic geometry/algebra/calculus GeoGebra system (http://www.geogebra.org) into the VMT environment this summer. The port of the open source GeoGebra code will enable it to function in a multi-user, synchronous online system. Integration into the VMT environment will support simultaneous text chat discussion of dynamic math diagrams, graphical referencing between chat and diagram, scrollable history of chat and diagrams, and pasting of diagrams into the associated wiki.
The integration of GeoGebra into the VMT environment will provide significant mathematical content and functionality to enhance mathematical exploration and communication. The integration includes plans to support importing and exporting of GeoGebra dynamic worksheets; this will allow teachers and students to take advantage of available curricular materials. The Math Forum plans to release the new system for worldwide usage, providing a convenient online venue for students to engage in synchronous collaborative learning within a rich environment for mathematical inquiry and knowledge-building interaction.

Name	**Tomas Recio**
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Title	**Intergeo: fostering the use of Dynamic Geometry Software in Europe**

Abstract

In this talk we will present a short account on the findings of an Intergeo survey on the situation of Dynamic Geometry Software (DGS) usage in Europe. We will also present a summary of the attempts to foster the establishment of communities of DGS users, through the different local user meetings that have been organized or attended by Intergeo members in the past year over a dozen European countries.

Intergeo is a EU funded, through eContentplus, project, see http://i2geo.net

(Abstract submitted to the Intergeo group)

Name	**Anna Takács Klingné**
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Title	**Educational experiences about using different computer programs**

Abstract

Traditionally the mathematical calculus course starts from the first semester at the University of Kaposvár. On the basis of our experiences it is too difficult or abstract for our students to pick up the elements of this subject. Among others it was motivated that we use computerized methods for the reason of demonstrations, visualization and better understanding of the concepts (monotonicity, extremal values, etc.) of the curriculum. Why we decided so that the Excel, Geogebra and Euler3D programs are convenient? Because of these programs are easy to avail and to use. Students are drawing graph of functions and sequences with Excel or/and Geogebra and recognizing their properties, and also using Excel Solver and Euler3d program to solve and demonstrate linear programming problems. In our presentation we give examples from students' tasks and opinions.

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Title	**European Computer Algebra Driving Licence**

Abstract

Goals of the ECADL do include a systematic overview of computer algebra software available to the students, to compare content of different packages and their applicability to real world problems.

Learning objectives
To require the students to learn how to use mathematical software to carry out tasks which are plainly most efficiently done by computer (such as solving large systems of simultaneous equations).
It is important that students do not just learn the relevant commands in the software package available to them. They must learn to use this packages discerningly, from a base of mathematical knowledge that will inform them, when the computer solution may be unreliable.

Reasoning for ECADL
The rapid changes in computer and information technology, the increase of computer power available and the complexity of mathematical software (Maple, Mathematica, MatLab, MuPad and Derive) now accessible to the students cannot be ignored; at the very least it demands a radical re-think on the way in which topics in the curriculum could be presented and does have an impact on the teaching of Mathematics in most of European universities.
There are two connected but distinct issues related to this expansion in the availability of computer and software, which are of considerable importance to the mathematics curriculum. The first is that new approaches to teaching and learning are made possible. The second is that enormously sophisticated mathematical software is now commonly available which allows the tackling of problems of such size and complexity that only a few years ago have been parts of research activities. Students require a personal knowledge of mathematics to be able to use mathematical software reliably and effectively.

Experience, results
The ECADL project is part Active Methods in teaching Mathematics and Informatics CEEEPUS Network CII-HU-028-03-09-10 (http://www.progmat.hu/ceepus), coordinated by the University of Miskolc, Hungary and it has been offered as part of a series of Computer algebra summer universities (2003-2006) where a number of 87 students have obtained the Computer Algebra Driving licence – basic level, and 15 students did the intermediate level course, and 3 the professional level.

Name	**János Karsai**
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Title	**Mathematica-aided study of impulsive systems in the math and applied classroom**

Abstract

A motion, for example, is described by a differential equation, but when some conditions are satisfied, suddenly an impulse changes the position and/or velocity of the moving body. Such systems are widely used in life sciences, medicine (like repeated dosing of drugs), control, etc. Although should, they are not considered in the frame of standard university courses. The reason is their formal study is much more complicated than that of either differential or difference equations. Computing and visualization tools are particularly important in both the theoretical study and in teaching them. The tools well known for ODE's and DE's are extended, as well as new functions are developed to study the aggregate effects of the continuous and discrete parts.
In the talk, we present the main functions of the packages developed in Mathematica 6 and 7 to solve and

visualize different types of impulsive systems. We also present some applications, demonstration projects for both math and applied classes, such as mechanical models, control, repeated drug administration, models in epidemiology and population dynamics, etc.
One can find some preliminary information on the web-page: http://silver.szote.u-szeged.hu/impulse.

Name	**Cezary Kaliszyk, Freek Wiedijk**
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Title	**Teaching logic using ProofWeb**

Abstract

We will present the system ProofWeb, that allows teaching logic using an interactive web interface to the proof assistant Coq. We start by showing a web interface to proof assistants.

We then present an extension for teaching, that makes the full power of Coq available to the students, but simultaneously presents the logic problems in a way that is customary in undergraduate logic courses.

We describe the supported natural deduction proofs styles, central tracking of the progress of the students, a database of logic problems, that also holds the students solutions and a parser that indicates whether the students used only the allowed inference rules of the logic.

Finally we show the possibilities of the use of the interface in collaborative proof development. We will describe our future project MathWiki that aims to create a wiki for formalized mathematics.

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Title	**Cognitive strategies and CAS**

Abstract

In our talk we will examine the following questions in relation to CAS use in mathematics teaching and learning:

- What kind of tools can CAS offer to enhance various cognitive strategies of students?
- How does CAS widens the available cognitive strategies and affect pragmatic, heuristic, and epistemic thinking schemes?
- How can CAS enhance meta-cognition and cognitive self-control/efficacy?
- How can CAS-enhanced curricula benefit from the examination of various cognitive strategies?
- How can the examination of CAS-related cognitive strategies assist modularisation of topics and thickening the "knowledge-representation web"?

Name	**Zsolt Lavicza**
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Title	**Mathematicians' views on CAS use in teaching**

Abstract

Technologies are becoming an integral part of everyday life and are increasingly used at all levels of education. Anecdotal evidence suggests an increase in the use of technology, particularly Computer Algebra

Systems (CAS), for the teaching and learning mathematics at universities. However, little is known about the current extent of CAS use in university-level teaching, the various practices mathematicians apply when teaching with technology, and mathematicians' views on the role of CAS in teaching mathematics. In my talk, I will outline results of a two-phase international comparative survey study, carried out in Hungary, the United Kingdom and United States, that examined 1) the extent of CAS use in; 2) the factors influencing CAS integration into; and 3) the effects of different teaching traditions on university-level CAS-assisted mathematics teaching. Results indicated that that many mathematicians extensively use CAS in their teaching and research. According to the developed statistical models, the teaching use of CAS was significantly influenced by mathematicians' CAS use in their own research and was largely affected by mathematicians' conceptions of the role of CAS in mathematical literacy. However, despite the substantial differences in teaching traditions in the participating countries the study did not reveal considerable differences in mathematicians' use and conceptions of CAS in their teaching between the participating countries.

Workshops

Title	**Beyond DGS - Simulations and Scripting with Cinderella**

Ulrich Kortenkamp, Pädagogische Hochschule Schwäbisch Gmünd, Germany
Andreas Fest, Technische Universität Berlin, Germany

Abstract

In this workshop we will explore the new facilities of the Interactive Geometry Software Cinderella.2 and work with physics simulations and custom scripts. For a sampler of what is possible see
http://www.mathe-vital.de
A basic knowledge of any geometry software is required. Participants will receive a temporary license for the software and can use their own laptops or the lab facilities at the conference.

Title	**"How to prepare interactive Mathematica demonstrations for classroom"** A microcourse and workshop

János Karsai, University of Szeged, Hungary

Abstract

One of the main improvement in Mathematica 6 is the dynamic functionality. Real-time graphics and functions for dynamic variables (Manipulate…) mean a breakthrough in the classroom usage of CAS systems. In the workshop, the participants can get acquainted with the elements of dynamic features of Mathematica and they learn how to prepare simple interactive demonstrations.
Colleagues are kindly encouraged and asked to send problems, topics for discussion in the frame of the workshop.

Title	Learning Math, Doing Math: Deductive Thinking and Construction Tasks with The Geometer's Sketchpad

A. Homero Flores, Math Teacher and researcher, Mexico

Abstract

A deductive thinking can be considered as the concatenation of ideas, each one determined by the previous one. In mathematics, deduction is the way in which we validate a conjecture, using general facts to justify less general or particular facts.
All individuals possess this way of constructing thoughts and jumping into conclusions, but we need to practice it in order to develop it deeper and get conclusion more educated, and so the decision we make will be the best ones at that moment.
In construction tasks with The Geometer's Sketchpad it is possible to explore the construction itself, make conjectures and try to validate them. When we try to explain why a construction works or why a conjecture is valid, we put ourselves in a theoretical context and make use of deduction as the means of getting the right answers.
When students work on a mathematical problem, they are used to think in the final result, not in the process they followed to get that final result. Most of the time, is in these processes were students find the justification of a conjecture and a review of the knowledge they already have.
In Learning Math, Doing Math, I propose, among many things, the fostering of a deductive thinking mainly –but not exclusively- with the development of construction tasks in Euclidian Geometry. An important part of the construction tasks is to ask students to explain how they did it and what suppositions they made and why. Now, the drag function of Sketchpad is of great help in exploring constructions in search of the right construction and validation of conjectures.
In this workshop, aimed at high school teachers and people interested in deductive thinking, we will go through the next activities to illustrate what I said in the previous paragraphs. They are a good way to introduce on the software and the teaching model activities. It is not mandatory that attendees have Sketchpad experience.
Activity 1
Construct a square using three different ways of doing it. In each case, explain why your construction is a square.
Actvity 2
Which kind of quadrilateral is the one that its diagonals meet each other at their midpoints? Explain your answer.
Construct the quadrilateral with Sketchpad. Write down step by step how the construction was made.

Title	**Learning More by Solving Series of Problems**

Martin Kollar, Comenius University, Bratislava , Slovakia
Jozef Hvorecky, Vysoka skola manazmentu, Bratislava, Slovakia

Abstract

Solving problems using a CAS-enabled calculator can be done at two hierarchical levels. The first one consists of solving isolated assignments. At the second level, students can start discussing relationships between those virtually unrelated problems. Results of those seemingly "haphazard" tasks may motivate them to form ideas, verify, modify, improve and generalize them. Using proper sets of problems, teachers then can lead their students to a sort of deeper knowledge – to creating hypotheses. The authors use the CASIO ClassPad calculators in secondary and early stages of tertiary Mathematics education and occasionally apply the above approach.
During our 60-minute hands-on workshop, we will demonstrate several outcomes of these educational experiments and explain in which fields of Mathematics they can be efficiently exploited. Sample areas are: L'Hospital's rule, instability of solutions of systems of linear equations, analytical geometry, animations in geometry, touching graphs, rolling geometric figures.
The workshop participants can play the role of learners. They are also invited to discuss and express their opinions on even more effective exploitation of this flexible tool.
No previous experience with CASIO ClassPad is needed.

Title	**WebMathematics Interactive 2** **A graphical user interface to computer algebra systems for students**

Zoltán Kovács, Bolyai Institute, University of Szeged, Hungary

Abstract

In 2007 the WMI2 Development Group announced a prerelease of WebMathematics Interactive 2 and showed the planned features on CADGME-2007. This web based, graphical Computer Algebra System (which is mostly a symbolic calculator) has been developed for more than two years by a 10 members team, and now is available in 10 languages (including English, German, French and Chinese). We reported our experiences in both programming and education practice in scientific forums and papers. This workshop will give a tutorial demonstration of the available functions in calculus, linear algebra, number theory, and polynomials. WMI2 has its demo web page on http://matek.hu.

Title	**How to implement automata constructions in Maple?**

George T. Maróti, University of Pécs, Hungary

Abstract

The aim of this workshop is to present a Maple oriented introduction to the package called aut which offers tools to construct, investigate, and visualize finite automata. The attribute

"Maple oriented" means we concentrate first of all on implementation instead of the exposition of automata theoretic notions and their properties. In this way automata theory is not our goal but the tool with which we demonstrate the usage of new types and construction tools offered by aut package.
We give examples for different automata constructions and show how these constructions are and new constructions can be implemented in Maple, which turns out as well as in the field of automata theory to be an extremely powerful tool in visualization and effectively helps us to understand the key features of abstract mathematical objects.

Title	**WIRIS, tools for web based learning environments (Moodle, Drupal, Joomla, Blogs)**

Ramon Eixarch, Maths for More, Barcelona, Spain

Abstract

In the last few years the use of web based tools as support to face to face teaching has grown rapidly. Those environments arise opportunities but many times also problems. The availability of user friendly tools but also powerful tools is crucial. WIRIS tools offer a global solution to mathematics content presentation on the web (equation editor), availability of web based math calculations (online CAS) and finally the use of assessment systems to evaluate students.
This workshop will focus on the possibilities that the use of CAS and DGS based tools offer to enhance your use of LMS (Learning management system). More emphasis will be made on the use of Moodle Open Source LMS.
Some related links
WIRIS www.wiris.com
WIRIS and Moodle www.wiris.com/moodle/
WIRIS tools in web tools www.wiris.com/documents/plugin.html

Title	**Modeling Towards Meaning of Functions**

Lilla Korenova, Comenius University, Bratislava, Slovakia
Jozef Hvorecky, Vysoka skola manazmentu, Bratislava, Slovakia

Abstract

The complete solution of real/life problems starts with the specification of the problem, its expression using mathematical concepts, solving it using a mathematical apparatus and interpreting its results using the terminology of the original problem area. In this four-stage process, graphing calculators can be efficiently used for speeding up its third ("purely mathematical") stage. The application of ICT will free the teachers' hands and allow them to concentrate on the pre-solution and post-solution relationships between the problem and its mathematical classification, representation, and meaning.
During our 60-minute hands-on workshop, this principle will be demonstrated on examples from financial mathematics and other real-life problems using CASIO ClassPad. The workshop participants can play the role of learners. They are also invited to discuss and express their opinions on even more effective exploitation of this flexible tool.

No previous experience with CASIO ClassPad is needed.

Title	**Coding Theory for the Classroom?**

Josef Böhm, ACDCA & Technical University of Vienna, Austria

Abstract

Workshop in the Frame of CADGME 2009.
The curriculum for the Austrian "Handelsakademie" (= College for Business Administration) does not contain only Cryptography" but also "Coding Theory". I took these chapters as one of the co-authors of a textbook series without knowing the intentions of the authors of the curriculum.
It was a challenge to find issues in Coding Theory which are beyond of only discussing the ASCII-Code and presenting ISBN- and EAN-Code and which are suitable for the students (age 17).
We will explain and work with data compressing like the Huffman-Code and with self correcting codes like the Hamming-Code. We will also show many questions which lead the students to a better understanding of the problems.
The workshop will comprise various classroom activities (answering questions of comprehension, short proofs, encoding and decoding by paper and pencil, encoding and decoding using provided tools, following instructions to create own tools, …). No previous knowledge is needed. CAS- and spreadsheet tools will support the activities.
Keywords: Coding Theory, Technology

Title	**Using M@th Desktop notebooks and palettes in the classroom**

Reinhard Simonovits, Handelsakademie Grazbachgasse, Austria
Philip Ramsden, Impercial College London, United Kingdom

Abstract

M@th Desktop (MD) is an interactive teaching and learning software package for mathematics classes. It contains a collection of Mathematica notebooks with associated palettes and packages, and is designed to be especially compatible with a blended learning strategy, in which computer - based activities are used to complement other approaches.
Universities and schools from eight European countries have been collaborating to develop MD – based courseware further together with didactical concepts for its use and material for in - service training. Two modules the MD Func and the MD Lin Alg were developed.
MD also provides tools for teachers and students to create their own palettes, notebooks and activities. We propose a 90 - minute workshop, to be led by Phil Ramsden and Reinhard Simonovits, with M@th Desktop as its themes.
These resources, whose target group is secondary schools in Europe, enable teachers to pursue a problem – based and group - oriented approach to teaching mathematics that emphasizes methods and concepts over computations.
Activities are designed to encourage an experimental, explorative approach. The resources consist of extensions to M@th Desktop, developed by Reinhard Simonovits.

The first 30 minutes will be presenter - led : colleagues will learn about the background to M@th Desktop; then, following a brief introduction to Mathematica, be introduced to the courseware and led on a tour of its main features.
The remainder of the session will be hands - on. Colleagues will mostly play the role of learners, undertaking some of the activities that the team have created, especially in the area of functions and graphs. Colleagues will have the opportunity to explore, and feed back on, the distinctive features of M@th Desktop, designed to make Mathematica' s power accessible to students who are not necessarily Mathematica experts.
Colleagues will also be invited to play the role of the teacher, and to reflect and feed back on possible modes of use of this courseware in the classroom, including the many opportunities that exist for customisation.

Title	**Interactive Geometry In Mathematica**

Phil Ramsden, Department of Mathematics, Imperial College London, United Kingdom

Abstract

Mathematica is now twenty years old. It has always been an environment for algebra, calculus, numerics and graphics, but since Version 6 was released two years ago, it has also offered users the ability to build real-time interactive content.
This facility is fully general; it applies across all mathematical domains. However, after a brief general introduction, we will focus on classical geometry, exploring ways of creating interactive geometrical figures that respond in real time to the user's actions.
There are two reasons for focusing on geometry. First, it will simply be interesting and fun to compare and contrast how this can be done in Mathematica with the way it is done in, say, Sketchpad or Cabri. Secondly, Mathematica's approach involves using a CAS-style structured language, and there are interesting pedagogic implications of this difference, which we will reflect on.
It may be possible to demonstrate new technology, not yet in any released version, though at the time of writing this cannot be guaranteed.

Title	**GeoGebra 3.2 – The new Spreadsheet View**

Yves Kreis, University of Luxembourg, Luxembourg;
Markus Hohenwarter, Florida State University, USA;
Judith Hohenwarter, Florida State University, USA

Abstract

GeoGebra (www.geogebra.org) is a free, multi-platform, open-source dynamic mathematics software with rapidly growing worldwide popularity. The basic idea of the software is to join dynamic geometry, algebra, and calculus, which other packages treat separately, into a single easy-to-use package for learning and

teaching mathematics from elementary through university level.
So far GeoGebra's user interface allowed algebraic (i.e. coordinates of points, equations) and graphical (i.e. points, function graphs) representations of mathematical objects. While coordinates and equations were displayed in the algebra view on the left hand side, their graphical representations could be seen in the graphics view on the right hand side.
The release version 3.2 (available in April 2009) adds another – third – view: the spreadsheet view. Now various kinds of mathematical objects – not only numbers – can also be put inside spreadsheet cells. All representations of the same object are linked dynamically and adapt automatically to changes made to any of the representations, no matter how they were initially created. This enables users to connect statistical reasoning with other areas of mathematics.
In this presentation we will introduce the new view, present some examples and discuss the opportunities it offers for the teaching and learning of mathematics.

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