Phase transitions of composition schemes: Mittag-Leffler and mixed Poisson distributions

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Abstract: Multitudinous probabilistic and combinatorial objects are associated with generating functions satisfying a composition scheme $F(z) = G(H(z))$. The analysis becomes challenging when this scheme is critical (i.e., G and H are simultaneously singular). Motivated by many examples (random mappings, planar maps, directed lattice paths), we consider a natural extension of this scheme, namely $F(z, u) = G(uH(z))M(z)$. We also consider a variant of this scheme, which allows us to analyse the number of H-components of a given size in F.

We prove that these two models lead to a rich world of limit laws, where we identify the key rôle played by a new universal three-parameter law: the beta-Mittag-Leffler distribution, which is essentially the product of a beta and a Mittag-Leffler distribution. We also prove (double) phase transitions, additionally involving Boltzmann and mixed Poisson distributions. In all cases we obtain moment convergence and local limit theorems. We present several applications of our results for, e.g., random walks, trees, Pólya urns, and the Chinese restaurant process.