Limit Theorems for Multivariate Discrete q-Distributions

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Abstract

Univariate discrete q-distributions based on stochastic models of sequences of independent Bernoulli trials with success probability varying geometrically either with the number of previous trials or with the number of previous successes or both with the number of previous trials and successes, have been introduced by Charalambides [2, 3, 4]. Analytically, he considered independent q-Bernoulli trials with success probability varying geometrically, with rate q, with the number of previous trials and defined the univariate q-binomial and negative q-binomial distributions of the first kind. Also, he considered independent q-Bernoulli trials with success probability varying geometrically, with rate q, with the number of previous successes and defined the univariate q-binomial and negative q-binomial distributions of the second kind. Moreover, he derived Heine distribution as direct discrete limit of the q-binomial distribution of the first kind and Euler distribution as a direct discrete limit of the q-binomial distribution of the second kind. Furthermore, he defined the univariate q-Pólya and inverse q-Pólya distributions by considering independent q-Bernoulli trials with success probability varying geometrically, with rate q, both with the number of previous trials and successes.

Univariate discrete q-distributions associated with q-orthogonal polynomials, have been considered by Kyriakoussis and Vamvakari [11, 13]. Analytically, they presented deformed types of the q-binomial and negative q-binomial distributions of the first kind and that of the Heine distribution and derived the associated q-orthogonal polynomials, based on q-Krawtchouk, q-Meixner and q-Charlier orthogonal polynomials respectively. Moreover, they established families of terminating and non-terminating q-Gauss hypergeometric series discrete distributions and associated them with classes of generalized q-Hahn and big q-Jacobi orthogonal polynomials, respectively. Also, they considered deformed types of the q-binomial and negative q-binomial distributions of the second kind and of the Euler distribution and derived the associated q-orthogonal polynomials, based on affine q-Krawtchouk, little q-Jacobi and little q-Laguerre/Wall polynomials respectively.

Recently, Vamvakari [17] introduced multivariate discrete q-distributions. Specifically, she derived a multivariate absorption distribution as a conditional distribution of a Heine process at a finite sequence of q-points in a time interval which had been defined by Kyriakoussis and Vamvakari [15]. Also, she deduced a multivariate q-hypergeometric distribution, as a conditional distribution of the multivariate absorption distribution.

Then, Charalambides [5, 6] introduced in detail q-multinomial, negative q-multinomial, multivariate q–Pólya and inverse q–Pólya distributions and also examined their limiting discrete distributions. Analytically, he considered a stochastic model of a sequence of independent Bernoulli trials with chain-composite successes (or failures), where the odds of success of a certain kind at a trial is assumed to vary geometrically, with rate q, with the number of previous trials and introduced the q-multinomial and negative q-multinomial distributions of the first kind as well as their discrete limit, multivariate Heine distribution. Also, he considered a stochastic model of a sequence of independent Bernoulli trials with chain-composite successes (or failures), where the probability of success of a certain kind at a trial varies geometrically, with rate q, with the number of previous successes and introduced the q-multinomial and negative q-multinomial distributions of the second kind kind as well as their discrete limit, multivariate Euler distribution.

Kyriakoussis and Vamvakari [12, 14, 16] studied the continuous limiting behaviour of the univariate discrete q-distributions. Analytically, they established the pointwise convergence of the q-binomial and the negative q-binomial distributions of the first kind, as well as of the Heine distribution, to a deformed Stieltjes Wigert continuous one. Moreover, they proved the pointwise convergence of the q-binomial and the negative q-binomial distributions of the second kind, as well as of the Euler distribution, to a deformed Gaussian one.

The aim of this work is to study the continuous limiting behaviour of multivariate discrete q-distributions. Limit theorems for q-multinomial and negative q-multinomial distributions are established. Specifically, the pointwise convergence of the q-multinomial and negative q-multinomial distributions of the first kind, as well as for their discrete limit, the multivariate Heine distribution, to a multivariate Stieltjes-Wigert type distribution, are proved. Also, the pointwise convergence of the q-multinomial and negative q-multinomial distributions of the second kind, as well as for their discrete limit, the multivariate Euler distribution, to a multivariate deformed Gaussian one, are provided. Furthermore, computer realizations of these approximations are presented, by providing some numerical calculations, using the *Mathematica* package HYPQ, indicating quite strong convergences.

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