Congruences for k-elongated partition diamonds

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Abstract: In 2007, George Andrews and Peter Paule published the eleventh paper in their series on MacMahon's partition analysis, with a particular focus on broken k-diamond partitions. On the way to broken k-diamond partitions, Andrews and Paule introduced the idea of k-elongated partition diamonds. Recently, Andrews and Paule revisited the topic of k-elongated partition diamonds in a paper that recently appeared in the Journal of Number Theory. Using partition analysis and the Omega operator, they proved that the generating function for the partition numbers $d_k(n)$ produced by summing the links of k-elongated plane partition diamonds of length n is given by $\frac{(q^2;q^2)_\infty^k}{(q;q)_\infty^{3k+1}}$ for each $k \geq 1$. A significant portion of their recent paper involves proving several congruence properties satisfied by d_1, d_2 and d_3, using modular forms as their primary proof tool. Since then, Nicolas Smoot has extended the work of Andrews and Paule, refining one of their conjectures and proving an infinite family of congruences modulo arbitrarily large powers of 3 for the function d_2.

In this work, our goal is to extend some of the results proven by Andrews and Paule in their recent paper by proving infinitely many congruence properties satisfied by the functions d_k for an infinite set of values of k. The proof techniques employed are all elementary, relying on generating function manipulations and classical q-series results.

This is joint work with Robson da Silva of Universidade Federal de Sao Paulo and Mike Hirschhorn of the University of New South Wales.