The calculus of series-divisors

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Abstract: The recurrence

$$np(n) = \sum_{i=1}^{n} \sigma(i)p(n-i)$$

has been used by Erdős and credited to Ford (1931) but appears in Ramanujan's notebooks. Here the divisor function $\sigma(n)$ is related to the partition function $p(n)$. The objective of our talk is to develop a calculus by which, virtually by inspection, one can find analogous recurrences connecting a divisor function with a partition-theoretic function. In particular, we associate each of Glaisher's 1885 list of divisor functions with an infinite product, which further suggests connections with partition-theoretic objects. From one of these recurrences, we obtain an elementary proof of Ramanujan's famous congruences $p(5n+4) \equiv 0 \pmod 5$ and $\tau(5n+5) \equiv 0 \pmod 5$. The proof requires no more than what Euler and Jacobi knew. The proof extends to embed the congruences into 4 infinite families of congruences for rational powers of the eta function. Many congruences of this nature have been found recently by Chan and Wang (2019); seven of their assertions are covered in our list. This is joint work with Hartosh Singh Bal