

Table 1: List of the main functions of the package ORE Morphisms

Morphisms(ConstCoeff,Rat)	Compute matrices P 's which define elements of $\text{hom}_D(M, M')$, i.e., which are such that there exist matrices Q 's satisfying $RP = QR$
Idempotents(ConstCoeff,Rat)	Compute matrices P 's defining idempotent elements of $\text{end}_D(M)$, i.e., which are such that there exist three matrices Q 's, Z 's and Z' 's satisfying $RP = QR$, $P^2 = P + ZR$, $Q^2 = Q + RZ + Z'R_2$, where $\ker_D(.R) = D^{1 \times r} R_2$
IdempotentsMat(ConstCoeff,Rat)	Compute idempotent matrices P 's defining idempotent elements of $\text{end}_D(M)$, i.e., which are such that there exist matrices Q 's satisfying $RP = QR$, $P^2 = P$, $Q^2 = Q$ (R has full row rank)
Riccati(ConstCoeff,Rat)	Find solutions Λ 's of the algebraic Riccati equation $\Lambda R \Lambda + (P - I_p) \Lambda + \Lambda Q + Z = 0$, where the pair (P, Q) defines an idempotent element of $\text{end}_D(M)$, i.e., satisfies the relations $RP = QR$, $P^2 = P + ZR$, $Q^2 = Q + RZ$, with $Z \in D^{p \times q}$ (R has full row rank)
KerMorphism(Rat)	Compute the kernel of $f \in \text{hom}_D(M, M')$, i.e., compute $S \in D^{r \times p}$ and $X \in D^{s \times r}$ such that: $\ker f = (D^{1 \times r} S) / (D^{1 \times q} R) \cong D^{1 \times r} / (D^{1 \times s} X)$.
ImMorphism(Rat)	Compute the image of $f \in \text{hom}_D(M, M')$ defined by a pair of matrices (P, Q) , i.e., $\text{im } f = (D^{1 \times (p+q')} (P^T \quad R'^T)^T) / (D^{1 \times q'} R')$ by reducing the rows of the matrix $(P^T \quad R'^T)^T$ modulo the left D -module $D^{1 \times q'} R'$
CoimMorphism(Rat)	Compute the coimage of $f \in \text{hom}_D(M, M')$, i.e., compute a matrix $S \in D^{r \times p}$ such that $\text{coim } f = D^{1 \times p} / (D^{1 \times r} S)$
CokerMorphism(Rat)	Compute the cokernel of $f \in \text{hom}_D(M, M')$ defined by a pair of matrices (P, Q) , i.e., $\text{coker } f = D^{1 \times p'} / (D^{1 \times (p+q')} (P^T \quad R'^T)^T)$
TestInj(Rat)	Test whether or not $f \in \text{hom}_D(M, M')$ is injective
TestSurj(Rat)	Test whether or not $f \in \text{hom}_D(M, M')$ is surjective
TestIso(Rat)	Test whether or not $f \in \text{hom}_D(M, M')$ is a D -isomorphism

HeuristicReduction(Rat)	Compute a reduction of the matrix R , i.e., compute an equivalent matrix with a block-triangular form. Bases of the different free left D -modules are computed using heuristic methods
HeuristicDecomposition(Rat)	Compute a decomposition of the matrix R , i.e., compute an equivalent matrix with a block-diagonal form. Bases of the different free left D -modules are computed using heuristic methods