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[> restart:
[> with(OreModules):
[> with(linalg):
[>
[> A:=DefineOreAlgebra(diff=[d1,x1],diff=[d2,x2],polynom=[x1,x2]):
[>
[> R:=evalm([ [d2^2], [d1*d2] ]);

$$R := \begin{bmatrix} d2^2 \\ d1 \ d2 \end{bmatrix} \quad (1)$$

[> S:=Integrability(R,A,[eta,zeta1,zeta2]);

$$S := [d1 \ \zeta1 - d2 \ \zeta2, \eta \ d2^2 - \zeta1, \eta \ d1 \ d2 - \zeta2] \quad (2)$$

[> CC:=remove(has,S,eta);

$$CC := [d1 \ \zeta1 - d2 \ \zeta2] \quad (3)$$

[> SyzygyModule(R,A);

$$\begin{bmatrix} d1 & -d2 \end{bmatrix} \quad (4)$$

[> FreeResolution(R,A);

$$table\left(\left[1 = \begin{bmatrix} d2^2 \\ d1 \ d2 \end{bmatrix}, 2 = \begin{bmatrix} d1 & -d2 \end{bmatrix}, 3 = INJ(1)\right]\right) \quad (5)$$

[> TorsionElements(R,[eta],A);

$$\left[ \begin{bmatrix} \frac{\partial^2}{\partial x2^2} \theta_1(x1, x2) = 0 \\ \frac{\partial^2}{\partial x2 \partial x1} \theta_1(x1, x2) = 0 \end{bmatrix}, \begin{bmatrix} \theta_1(x1, x2) = \eta \end{bmatrix} \right] \quad (6)$$

[>
[> ext0_M:=Exti(R,A,0);

$$ext0\_M := INJ(1) \quad (7)$$

[> ext1_M:=Exti(R,A,1);

$$ext1\_M := \left[ \begin{bmatrix} d2 \end{bmatrix}, \begin{bmatrix} d2 & d1 \end{bmatrix}, \begin{bmatrix} -d1 \\ d2 \end{bmatrix} \right] \quad (8)$$

[> ext2_M:=Exti(R,A,2);

$$ext2\_M := \left[ \begin{bmatrix} d2 \\ d1 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix}, SURJ(1) \right] \quad (9)$$

[> ext3_M:=Exti(R,A,3);

$$ext3\_M := [undefined, ZERO, ZERO] \quad (10)$$

[>
[> R_trans:=transpose(R);

$$R\_trans := \begin{bmatrix} d2^2 & d1 \ d2 \end{bmatrix} \quad (11)$$

[> FreeResolution(R_trans,A);

$$table\left(\left[1 = \begin{bmatrix} d2^2 & d1 \ d2 \end{bmatrix}, 2 = INJ(1)\right]\right) \quad (12)$$


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$$\begin{array}{l} \text{> ext0_N:=Exti(R_trans,A,0);} \\ \text{ext0_N:=} \left[\begin{array}{cc} d1 & -d2 \end{array} \right] \end{array} \quad (13)$$

$$\begin{array}{l} \text{> ext1_N:=Exti(R_trans,A,1);} \\ \text{ext1_N:=} \left[\left[\begin{array}{cc} d2^2 & \\ d1 & d2 \end{array} \right], \left[\begin{array}{cc} 1 & \end{array} \right], SURJ(1) \right] \end{array} \quad (14)$$

$$\begin{array}{l} \text{> ext2_N:=Exti(R_trans,A,2);} \\ \text{ext2_N:=} [undefined, ZERO, ZERO] \end{array} \quad (15)$$

$\begin{array}{l} \text{>} \\ \text{>} \end{array}$