```
> restart:
> with(OreModules):
> with(linalg):
>
> A:=DefineOreAlgebra(diff=[dx,x],diff=[dy,y],diff=[dz,z],polynom=
  [x,y,z]):
>
> R:=evalm([[dx,0,0],[0,dy,0],[0,0,dz],[0,dz,dy],[dz,0,dx],[dy,dx,0]
  ]);
```

$$R := \begin{bmatrix} dx & 0 & 0 \\ 0 & dy & 0 \\ 0 & 0 & dz \\ 0 & dz & dy \\ dz & 0 & dx \\ dy & dx & 0 \end{bmatrix} \tag{1}$$

```
> var:=x,y,z:
> ApplyMatrix(R,[u(var),v(var),w(var)],A);
```

$$\begin{bmatrix} \frac{\partial}{\partial x} u(x, y, z) \\ \frac{\partial}{\partial y} v(x, y, z) \\ \frac{\partial}{\partial z} w(x, y, z) \\ \frac{\partial}{\partial z} v(x, y, z) + \frac{\partial}{\partial y} w(x, y, z) \\ \frac{\partial}{\partial z} u(x, y, z) + \frac{\partial}{\partial x} w(x, y, z) \\ \frac{\partial}{\partial y} u(x, y, z) + \frac{\partial}{\partial x} v(x, y, z) \end{bmatrix} \tag{2}$$

```
> DimensionRat(R,A);
```

$$0 \tag{3}$$

```
> B:=KBasis(R,A);
```

$$B := \left[\lambda_1, \lambda_2, \lambda_2\, dx, \lambda_3, \lambda_3\, dy, \lambda_3\, dx\right] \tag{4}$$

```
> nops(Basis);
```

$$1 \tag{5}$$

```
> R2:=SyzygyModule(R,A);
```

(6)

$$R2 := \begin{bmatrix} 2\,dz\,dy & 0 & 0 & dx^2 & -dx\,dy & -dx\,dz \\ dz^2 & 0 & dx^2 & 0 & -dx\,dz & 0 \\ dy^2 & dx^2 & 0 & 0 & 0 & -dx\,dy \\ 0 & dz^2 & dy^2 & -dz\,dy & 0 & 0 \\ 0 & -2\,dx\,dz & 0 & dx\,dy & -dy^2 & dz\,dy \\ 0 & 0 & 2\,dx\,dy & -dx\,dz & -dz\,dy & dz^2 \end{bmatrix} \tag{6}$$

```
> FreeResolution(R,A);
```

$$table \begin{matrix} dx & 0 & 0 \\ 0 & dy & 0 \\ 0 & 0 & dz \\ 0 & dz & dy \\ dz & 0 & dx \\ dy & dx & 0 \end{matrix} \tag{7}$$

```
>
>
> R:=evalm([[dx,0,0,0,dz,dy],[0,dy,0,dz,0,dx],[0,0,dz,dy,dx,0]]);
```

$$R := \begin{bmatrix} dx & 0 & 0 & 0 & dz & dy \\ 0 & dy & 0 & dz & 0 & dx \\ 0 & 0 & dz & dy & dx & 0 \end{bmatrix} \tag{8}$$

```
> OreRank(R,A);
```

$$3 \tag{9}$$

```
> R_adj:=Involution(R,A);
```

$$R\_adj := \begin{bmatrix} -dx & 0 & 0 \\ 0 & -dy & 0 \\ 0 & 0 & -dz \\ 0 & -dz & -dy \\ -dz & 0 & -dx \\ -dy & -dx & 0 \end{bmatrix} \tag{10}$$

```
> Ext1:=Exti(R_adj,A,1);
```

(11)

$$Ext1 := \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} dx & 0 & 0 & 0 & dz & dy \\ 0 & dy & 0 & dz & 0 & dx \\ 0 & 0 & dz & dy & dx & 0 \end{bmatrix}, [2\, dz\, dy, dz^2, dy^2, 0, 0, 0], [0, 0, dx^2, dz^2, -2\, dx\, dz, 0], [0, dx^2, 0, dy^2, 0, 2\, dx\, dy], [dx^2, 0, 0, -dz\, dy, dx\, dy, -dx\, dz], [-dx\, dy, -dx\, dz, 0, 0, -dy^2, -dz\, dy], [-dx\, dz, 0, -dx\, dy, 0, dz\, dy, dz^2] \right] \quad (11)$$

```
> Q:=Ext1[3];
```

$$Q := \begin{bmatrix} 2\, dz\, dy & dz^2 & dy^2 & 0 & 0 & 0 \\ 0 & 0 & dx^2 & dz^2 & -2\, dx\, dz & 0 \\ 0 & dx^2 & 0 & dy^2 & 0 & 2\, dx\, dy \\ dx^2 & 0 & 0 & -dz\, dy & dx\, dy & -dx\, dz \\ -dx\, dy & -dx\, dz & 0 & 0 & -dy^2 & -dz\, dy \\ -dx\, dz & 0 & -dx\, dy & 0 & dz\, dy & dz^2 \end{bmatrix} \quad (12)$$

```
>
> Ext2:=Exti(R_adj,A,2);
```

$$Ext2 := \left[\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2\, dz\, dy & dz^2 & dy^2 & 0 & 0 & 0 \\ dx\, dz & 0 & dx\, dy & 0 & -dz\, dy & -dz^2 \\ dx\, dy & dx\, dz & 0 & 0 & dy^2 & dz\, dy \\ dx^2 & 0 & 0 & -dz\, dy & dx\, dy & -dx\, dz \\ 0 & dx^2 & 0 & dy^2 & 0 & 2\, dx\, dy \\ 0 & 0 & dx^2 & dz^2 & -2\, dx\, dz & 0 \end{bmatrix}, [-dz, dy, 0], [\right. \quad (13)$$

$$2\,dy, 0, 0], [0, -2\,dz, 0], [0, 0, -2\,dx], [0, -dx, -dz], [-dx, 0, dy]]$$

```
> Ext3:=Exti(R_adj,A,3);
```

$$Ext3 := \left[\left[\begin{array}{ccc} dy & 0 & 0 \\ dz^2 & 0 & 0 \\ dx\,dz & 0 & 0 \\ dx^2 & 0 & 0 \\ 0 & dz & 0 \\ 0 & dy^2 & 0 \\ 0 & dx\,dy & 0 \\ 0 & dx^2 & 0 \\ 0 & 0 & dx \\ 0 & 0 & dz^2 \\ 0 & 0 & dz\,dy \\ 0 & 0 & dy^2 \end{array}\right], \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right], SURJ(3)\right] \tag{14}$$

```
> Ext4:=Exti(R_adj,A,4);
```

$$Ext4 := [undefined, ZERO, ZERO] \tag{15}$$

```
>
> Q_Morera:=submatrix(Q,1..6,[1,5,6]);
```

$$Q\_Morera := \left[\begin{array}{ccc} 2\,dz\,dy & 0 & 0 \\ 0 & -2\,dx\,dz & 0 \\ 0 & 0 & 2\,dx\,dy \\ dx^2 & dx\,dy & -dx\,dz \\ -dx\,dy & -dy^2 & -dz\,dy \\ -dx\,dz & dz\,dy & dz^2 \end{array}\right] \tag{16}$$

```
> SyzygyModule(Q_Morera,A);
```

$$\left[\begin{array}{cccccc} dx & 0 & 0 & 0 & dz & dy \\ 0 & dy & 0 & dz & 0 & dx \\ 0 & 0 & dz & dy & dx & 0 \end{array}\right] \tag{17}$$

```
> Q_Maxwell:=submatrix(Q,1..6,[2,3,4]);
```

$$Q\_Maxwell := \begin{bmatrix} dz^2 & dy^2 & 0 \\ 0 & dx^2 & dz^2 \\ dx^2 & 0 & dy^2 \\ 0 & 0 & -dz\,dy \\ -dx\,dz & 0 & 0 \\ 0 & -dx\,dy & 0 \end{bmatrix} \tag{18}$$

```
> SyzygyModule(Q_Maxwell,A);
```

$$\begin{bmatrix} dx & 0 & 0 & 0 & dz & dy \\ 0 & dy & 0 & dz & 0 & dx \\ 0 & 0 & dz & dy & dx & 0 \end{bmatrix} \tag{19}$$

```
> Q_adj:=Involution(Q,A);
```

$$Q\_adj := \begin{bmatrix} 2\,dz\,dy & 0 & 0 & dx^2 & -dx\,dy & -dx\,dz \\ dz^2 & 0 & dx^2 & 0 & -dx\,dz & 0 \\ dy^2 & dx^2 & 0 & 0 & 0 & -dx\,dy \\ 0 & dz^2 & dy^2 & -dz\,dy & 0 & 0 \\ 0 & -2\,dx\,dz & 0 & dx\,dy & -dy^2 & dz\,dy \\ 0 & 0 & 2\,dx\,dy & -dx\,dz & -dz\,dy & dz^2 \end{bmatrix} \tag{20}$$

```
> simplify(evalm(Q_adj-R2));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

```
>
```