```
> restart:
> with(OreModules):
> with(linalg):
>
> A:=DefineOreAlgebra(diff=[d,t],polynom=[t]):
>
> R:=evalm([[-t^2,t*d-1],[-(t*d+2),d^2]]);
```

$$R := \begin{bmatrix} -t^2 & t\,d-1 \\ -t\,d-2 & d^2 \end{bmatrix} \tag{1}$$

```
> LeftInverse(R,A);
```

$$[\,] \tag{2}$$

```
> RightInverse(R,A);
```

$$[\,] \tag{3}$$

```
> F:=FreeResolution(R,A);
```

$$F := table\left(\left[1 = \begin{bmatrix} -t^2 & t\,d-1 \\ -t\,d-2 & d^2 \end{bmatrix}, 2 = \begin{bmatrix} d & -t \end{bmatrix}, 3 = INJ(1)\right]\right) \tag{4}$$

```
> R2:=evalm(F[2]);
```

$$R2 := \begin{bmatrix} d & -t \end{bmatrix} \tag{5}$$

```
> S2:=RightInverse(R2,A);
```

$$S2 := \begin{bmatrix} t \\ d \end{bmatrix} \tag{6}$$

```
> F:=Involution(evalm(1-Mult(S2,R2,A)),A);
```

$$F := \begin{bmatrix} t\,d+2 & -d^2 \\ t^2 & -t\,d+1 \end{bmatrix} \tag{7}$$

```
> S_adj:=Factorize(F,Involution(R,A),A);
```

$$S\_adj := \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{8}$$

```
> S:=Involution(S_adj,A);
```

$$S := \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{9}$$

```
> Verif:=simplify(evalm(Mult(S2,R2,A)+Mult(R,S,A)));
```

$$Verif := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

```
> GenInv:=GeneralizedInverse(R,A);
```

$$GenInv := \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \tag{11}$$

```
> simplify(evalm(Mult(R,S,R,A)-R));
```

(12)

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

```
> R_adj:=Involution(R,A);
```

$$R\_adj := \begin{bmatrix} -t^2 & t\,d-1 \\ -t\,d-2 & d^2 \end{bmatrix} \tag{13}$$

```
> Ext1:=Exti(R_adj,A,1);
```

$$Ext1 := \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -t^2 & t\,d-1 \\ -t\,d-2 & d^2 \end{bmatrix}, \begin{bmatrix} -d \\ -t \end{bmatrix} \right] \tag{14}$$

```
> T:=LeftInverse(Ext1[3],A);
```

$$T := \begin{bmatrix} t & -d \end{bmatrix} \tag{15}$$

```
> Q:=Parametrization(R,A);
```

$$Q := \begin{bmatrix} -\frac{\mathrm{d}}{\mathrm{d}t}\,\xi_1(t) \\ -t\,\xi_1(t) \end{bmatrix} \tag{16}$$

```
> evalm([[y[1]],[y[2]]])=evalm(Q);
```

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -\frac{\mathrm{d}}{\mathrm{d}t}\,\xi_1(t) \\ -t\,\xi_1(t) \end{bmatrix} \tag{17}$$

```
> ApplyMatrix(R,Q,A);
```

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

```
> xi[1]:=ApplyMatrix(T,[y[1](t),y[2](t)],A)[1,1];
```

$$\xi_1 := t\,y_1(t) - \left(\frac{\mathrm{d}}{\mathrm{d}t}\,y_2(t) \right) \tag{19}$$

```
>
```