```
> restart:
> with(OreModules):
> with(linalg):
>
> A:=DefineOreAlgebra(diff=[d1,x1],diff=[d2,x2],diff=[d3,x3],
  polynom=[x1,x2,x3]):
>
> R:=evalm([[d1,d2,d3]]);
```

$$R := \begin{bmatrix} d1 & d2 & d3 \end{bmatrix} \tag{1}$$

```
> F:=FreeResolution(R,A);
```

$$F := table\left(\left[1 = \begin{bmatrix} d1 & d2 & d3 \end{bmatrix}, 2 = INJ(1)\right]\right) \tag{2}$$

```
> R_adj:=Involution(R,A);
```

$$R\_adj := \begin{bmatrix} -d1 \\ -d2 \\ -d3 \end{bmatrix} \tag{3}$$

```
> G:=FreeResolution(R_adj,A);
```

$$G := table\left(\left[1 = \begin{bmatrix} -d1 \\ -d2 \\ -d3 \end{bmatrix}, 2 = \begin{bmatrix} -d3 & 0 & d1 \\ -d2 & d1 & 0 \\ 0 & -d3 & d2 \end{bmatrix}, 3 = \begin{bmatrix} -d2 & d3 & d1 \end{bmatrix}, 4 = INJ(1)\right]\right) \tag{4}$$

```
> Ext1:=Exti(R_adj,A,1);
```

$$Ext1 := \left[\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} d1 & d2 & d3 \end{bmatrix}, \begin{bmatrix} d3 & d2 & 0 \\ 0 & -d1 & d3 \\ -d1 & 0 & -d2 \end{bmatrix}\right] \tag{5}$$

```
> Ext2:=Exti(R_adj,A,2);
```

$$Ext2 := \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} d3 & d2 & 0 \\ d1 & 0 & d2 \\ 0 & d1 & -d3 \end{bmatrix}, \begin{bmatrix} d2 \\ -d3 \\ -d1 \end{bmatrix}\right] \tag{6}$$

```
> Ext3:=Exti(R_adj,A,3);
```

$$Ext3 := \left[\begin{bmatrix} d3 \\ d2 \\ d1 \end{bmatrix}, \begin{bmatrix} 1 \end{bmatrix}, SURJ(1)\right] \tag{7}$$

```
> Ext4:=Exti(R_adj,A,4);
```

$$Ext4 := [undefined, ZERO, ZERO] \tag{8}$$

```
> P:=Ext1[3];
```

$$P := \begin{bmatrix} d3 & d2 & 0 \\ 0 & -d1 & d3 \\ -d1 & 0 & -d2 \end{bmatrix} \tag{9}$$

```
> x:=x1,x2,x3:
> eta:=ApplyMatrix(P,[eta1(x),eta2(x),eta3(x)],A);
```

$$\eta := \begin{bmatrix} \frac{\partial}{\partial x3}\,\eta 1(x1,x2,x3) + \frac{\partial}{\partial x2}\,\eta 2(x1,x2,x3) \\ -\left(\frac{\partial}{\partial x1}\,\eta 2(x1,x2,x3)\right) + \frac{\partial}{\partial x3}\,\eta 3(x1,x2,x3) \\ -\left(\frac{\partial}{\partial x1}\,\eta 1(x1,x2,x3)\right) - \left(\frac{\partial}{\partial x2}\,\eta 3(x1,x2,x3)\right) \end{bmatrix} \tag{10}$$

```
> ApplyMatrix(R,eta,A);
```

$$\begin{bmatrix} 0 \end{bmatrix} \tag{11}$$

```
> OreRank(R,A);
```

$$2 \tag{12}$$

```
> Min:=MinimalParametrizations(R,A);
```

$$Min := \left[\begin{bmatrix} d3 & d2 \\ 0 & -d1 \\ -d1 & 0 \end{bmatrix}, \begin{bmatrix} d3 & 0 \\ 0 & d3 \\ -d1 & -d2 \end{bmatrix}, \begin{bmatrix} d2 & 0 \\ -d1 & d3 \\ 0 & -d2 \end{bmatrix} \right] \tag{13}$$

```
> map(a->SyzygyModule(a,A),Min);
```

$$\left[\begin{bmatrix} d1 & d2 & d3 \end{bmatrix}, \begin{bmatrix} d1 & d2 & d3 \end{bmatrix}, \begin{bmatrix} d1 & d2 & d3 \end{bmatrix} \right] \tag{14}$$

```
>
```