Theorem and Algorithm Checking for Courses on Logic and Formal Methods

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The RISC Algorithm Language (RISCAL) is a language for the formal modeling of theories and algorithms. A RISCAL specification describes an infinite class of models each of which has finite size; this allows to fully automatically check in such a model the validity of all theorems and the correctness of all algorithms. RISCAL thus enables us to quickly verify/falsify the specific truth of propositions in sample instances of a model class before attempting to prove their general truth in the whole class: the first can be achieved in a fully automatic way while the second typically requires our assistance. RISCAL has been mainly developed for educational purposes. To this end this paper reports on some new enhancements of the tool: the automatic generation of checkable verification conditions from algorithms, the visualization of the execution of procedures and the evaluation of formulas illustrating the computation of their results, and the generation of Web-based student exercises and assignments from RISCAL specifications. Furthermore, we report on our first experience with RISCAL in the teaching of courses on logic and formal methods and on further plans to use this tool to enhance formal education.

1 Introduction

Teaching courses that rely on logic-based formalisms (such as the specification and verification of computer programs) is hampered by the difficulty of conveying to students the meaning of logic formulas respectively the subtly different meanings of multiple variants of logic formulations. This is especially the case if the effects of such formulations are only exhibited by the consequent provability of propositions (such as verification conditions whose validity implies the correctness of algorithms). The use of a tool that automates a proving calculus is here typically of little help: first, since such a tool (in a sufficiently rich logic) only provides semi-decisions, a conjecture may still be correct, even if it cannot be proved by the tool; second, even if a conjecture is indeed wrong, a failed proof attempt typically does not shed much light on the core reason. This is the more deplorable since in practice (as well in education as in research) most time is actually spent in trying to prove conjectures that do not hold. This is also the fundamental limitation on the educational use of tools for deductive program verification, be they “interactive”, “automatic”, or “auto-active”; prominent examples of such tools are the interactive KeY verifier \cite{key}, the verification platform Why3 \cite{why3} with various interactive and automatic backends, the OpenJML tool with various SMT solvers as automatic backends \cite{openjml}, and the auto-active systems Spec# \cite{spec} and Dafny \cite{dafny}, both based on the Boogie backend.

One attempt to overcome these problems is to use reasoning tools that are based on model checking \cite{modelchecking} rather than proving: first, such tools generally provide full decisions (if the violation of a conjecture is reported, the conjecture indeed does not hold); second, if a conjecture does not hold, such

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*Supported by the Johannes Kepler University Linz, Linz Institute of Technology (LIT), Project LOGTECHEDU “Logic Technology for Computer Science Education” and by the OEAD WTZ project SK 14/2018 SemTech.
tools provide a counter-example that may help to understand the underlying reason. The downside of this ability of model checkers is the necessity to restrict the domain of investigation to models of finite size whose properties are consequently decidable. Furthermore, since model checkers have arisen in the context of the verification of hardware/software systems, they typically only support specification languages of limited expressiveness, usually based on a quantifier-free logic (propositional logic, propositional linear temporal logic, or some other logic where the satisfiability problem is decidable); if also quantifiers are supported, checkers typically apply heuristics and thus loose decidability, yielding the same disadvantages that plague automated provers. This makes these tools less suitable for the formal education of students of computer science or mathematics.

The RISC Algorithm Language (RISCAL) \[19, 14\] is an attempt to make model checking an attractive option also for such educational scenarios. RISCAL does not at all compromise the expressiveness of its specification language: it supports full (first-order) predicate logic on top of a rich basis of types (integers, tuples, records, arrays, sets, recursive types) with corresponding operations. To make the resulting theory still decidable, the only restriction is that all types have bounds that make the model finite: e.g., $\mathbb{Z}[N,M]$ denotes the type of all integers in interval $[N,M]$ while $\text{Array}[N,T]$ denotes the type of all arrays of length $N$ whose values have type $T$. However, these type bounds may be unspecified constants, thus each RISCAL specification denotes an infinite class of models where each instance of this class is finite. By choosing some values for these constants we may pick an instance of this class and check all its properties that are expressible in the logic. The basic technique to check the validity of a formula (and in general of every phrase in RISCAL such as terms and commands) is to evaluate its semantics; for instance to evaluate a formula $(\forall x: T. F)$ we enumerate all values of type $T$, bind variable $x$ to each of these values in turn and evaluate the formula body $F$ in this binding. Since the language also supports constructs such as $(\text{choose} x: T. F)$ which denotes an arbitrary value $x$ of type $T$ that satisfies formula $F$, the semantics of RISCAL is in general multi-valued: if we select for the model checker a “nondeterministic” evaluation mode, all possible outcomes are computed (in a lazy fashion); in its (faster) “deterministic” mode, only one of the outcomes is considered.

In predecessor publications we have already described the basic capabilities of RISCAL \[22, 20\]. The purpose of this paper is to outline some recent enhancements that target the usefulness of RISCAL in educational scenarios: the automatic generation of checkable verification conditions, the visualization of procedure execution and formula evaluation, and the generation of Web-based student exercises respectively assignments from RISCAL specifications. Furthermore, we give first reports on our experience of the use of RISCAL in the classroom and sketch future plans.

RISCAL is not the first attempt to apply model checking technology in formal modeling: various other modeling languages support checking respectively counterexample generation \[9\], differing from RISCAL mainly in the application domain of the language and/or the exhaustiveness of the analysis. The development of RISCAL has been very much inspired by the TLA Toolbox which incorporates a model checker for a subset of TLA+ \[11\]; also the use of unspecified constants as model bounds has been inspired by this toolkit. However, TLA+ is untyped and actually also supports models of infinite size; thus the toolkit does not really represent a reliable decision procedure for TLA+ specifications.

The specification language Alloy \[10\] is based on a relational logic designed for modeling the states of abstract systems and the executions of such systems; given bounds for state sizes and execution lengths, the Alloy Analyzer finds all executions leading to states satisfying/violating given properties and thus indeed represents a decision procedure. However, the specification language of Alloy is quite system-oriented which makes it not very attractive for describing general mathematical domains and algorithms \[15\]. On the other hand, the counterexample generator Nitpick \[3\] for the proof assistant Isabelle/HOL supports classical predicate logic but may (due to the presence of unbounded quantifiers)
fail to find counterexamples; in an “unsound mode” quantifiers are artificially bounded, but then invalid counterexamples may be reported.

While Alloy and Nitpick are based on SAT-solving techniques, RISCAL’s implementation is actually closer to that of the test case generator Smallcheck [16]. This system generates for the parameters of a Haskell function in an exhaustive way argument values up to a certain bound; the results of the corresponding function applications may be checked against properties expressed in first-order logic (encoded as executable higher-order Haskell functions). However, unlike RISCAL, the value generation bound is specified by global constants rather than by the types of parameters and quantified variables such that separate mechanisms have to be used to restrict searches for counterexamples.

The remainder of this paper is organized as follows: To make the presentation self-contained, we give in Section 2 (based on previously published material) a short overview on the core functionality of RISCAL. In Section 3 we discuss new (not yet formally published) features of RISCAL. In Section 4 we discuss our experience with and plans for the use of RISCAL in education. Section 5 presents our conclusions and sketches ideas for the future.

2 The RISCAL System

To make this paper self-contained, we give a short summary overview on RISCAL with material taken from [22, 20]; a more detailed tutorial on the use of the software and the complete reference of its specification language can be found in the manual [19].

Figure 1 depicts the user interface of the RISCAL software with an editor frame for RISCAL specifications on the left and the control widgets and output frame of the checker on the right. The RISCAL specification language is based on a statically typed variant of first order predicate logic. On the one hand, it allows to develop mathematical theories such as that of the greatest common divisor depicted at the top of Figure 2; on the other hand, it also enables the specification of algorithms such as Euclid’s algorithm...
val N: N; type nat = N[N];

pred divides(m:nat,n:nat) ⇔ ∃p:nat. m · p = n;
fun gcd(m:nat,n:nat): nat
  requires m ≠ 0 ∨ n ≠ 0;
  = choose result:nat with
    divides(result,m) ∧ divides(result,n) ∧ ¬∃r:nat. divides(r,m) ∧ divides(r,n) ∧ r > result;
theorem gcd0(m:nat) ⇔ m ≠ 0 ⇒ gcd(m,0) = m;
theorem gcd1(m:nat,n:nat) ⇔ m ≠ 0 ∨ n ≠ 0 ⇒ gcd(m,n) = gcd(n,m);
theorem gcd2(m:nat,n:nat) ⇔ 1 ≤ n ∧ n ≤ m ⇒ gcd(m,n) = gcd(m%n,n);
proc gcdp(m:nat,n:nat): nat
  requires m ≠ 0 ∨ n ≠ 0;
  ensures result = gcd(m,n);
{ var a:nat := m; var b:nat := n;
  while a > 0 ∧ b > 0 do
    invariant a ≠ 0 ∨ b ≠ 0;
    invariant gcd(a,b) = gcd(old_a,old_b);
    decreases a+b;
    { if a > b then a := a%b; else b := b%a;
    }
  return if a = 0 then b else a;
}

Figure 2: Euclid’s Algorithm in RISCAL
This ensures that for all possible parameter values, the truth value of the corresponding formula is "true". Likewise, we can validate that the procedure gcdp satisfies its specification:

```
Executing gcdp(Z,Z) with all 441 inputs.
Execution completed for ALL inputs (933 ms, 440 checked, 1 inadmissible).
```

This check indeed evaluates the procedure specification and the embedded loop annotations; here one input (that with \( m = n = 0 \)) is reported as "inadmissible" which indicates that it violates the precondition of the procedure; consequently, the procedure need not be checked with that input.

### 3 New Features of RISCAL

In this section, we report on new features of RISCAL relevant for its application in the context of educational scenarios; these features have been described in a series of technical reports \[19, 23, 24, 21\] but not yet been formally published before.

#### 3.1 Generation and Checking of Verification Conditions

To further validate the correctness of algorithms (and as a prerequisite of subsequent general correctness proofs), RISCAL provides since version 2.0 a verification condition generator (VCG). This VCG produces from the definition of an operation (including its specification and other formal annotations) a set of verification conditions, i.e., theorems whose validity implies that, for all arguments that satisfy the precondition of the operation, its execution terminates and produces a result that satisfies its postcondition. These theorems are plain RISCAL formulas; their validity (in a finite domain) can be automatically checked within RISCAL itself.

This VCG is fundamentally based on Dijkstra’s weakest precondition (wp) calculus. In order to prove the correctness of a procedure

```
proc p(x:T1):T2 requires P(x); ensures Q(x,result) { C; return r; }
```

we generate a theorem

```
theorem _p_Corr(x:T1) requires P(x); ⇔ wp(C, let result = r in Q);
```

where \( wp(C,Q) \) is a formula generated from the body \( C \) of the procedure and its postcondition \( Q \) (called “the weakest precondition of \( C \) with respect to \( Q \)”). In a nutshell, the theorem states that every argument \( x \) that satisfies the precondition \( P \) of the procedure also satisfies that weakest precondition (however, as discussed below, we actually generate not only one such big formula but a lot of smaller ones).

Generating and checking verification conditions in RISCAL serves a particular purpose: it not only ensures that the algorithm works as expected (this has been demonstrated already by checking the algorithm) but also establishes that all annotations (specifications, loop invariants, termination terms) are strong enough to verify the correctness of the algorithm by formal proof. If checking the verification conditions succeeds, this indeed demonstrates that such a proof is possible in the particular model (determined by the concrete values assigned to the type bounds) in which the check has taken place. However, the success of such a check gives also reason to believe (at least increases our confidence) that such a proof may be possible for the infinite class of all possible models (i.e., for all infinitely many possible values of the type bounds). At least, if a check fails, we know that there is no point in attempting such a general proof before the error exhibited in the particular model has been fixed.

Our default assumption is that (due to some problem in the definition of the operation respectively in its specification or annotations) a correctness proof will fail. Therefore RISCAL tries to help the user to find the reason of such a failure by the following strategy:
Rather than generating a single big verification condition whose validity ensures the overall correctness of the algorithm, RISCAL generates a lot of smaller verification conditions each of which demonstrates some aspect of correctness. Thus, if a particular condition fails, we do not only know that the overall correctness of the algorithm cannot be established but we can focus on that aspect of correctness expressed by the corresponding condition.

Each verification condition is linked to those parts of the algorithm from which the condition has been generated and that are therefore relevant for its validity. These parts are visualized in the editor such that we get a quick intuition on which parts of the algorithm we have to concentrate.

In case of the greatest common divisor procedure \texttt{gcdp} that was sketched in Section 2, RISCAL generates the verification conditions listed in Figure 3. These conditions are initially indicated as red tasks; having successfully checked them in RISCAL (by double clicking the tasks or right-clicking the task and selecting in the pop-up menu the entry “Execute Task”), the tasks turn blue. These tasks check whether the result is correct (whether the precondition of the procedure implies the core of the weakest precondition of its body with respect to the specified postcondition), whether the loop invariant initially holds, whether the loop measure is non-negative, whether the loop invariant is preserved, whether the loop measure is decreased, and whether the preconditions of the various operations hold.

For instance, the task “Is the result correct?” checks the validity of the following verification condition (whose definition can be displaced by clicking a corresponding entry in a popup menu):

\begin{verbatim}
theorem _gcdp_S_CorrOp0(m:nat, n:nat)
requires (m \neq 0) \lor (n \neq 0);
\text{let} a = m \text{ in } (\text{let} b = n \text{ in } (\forall a:nat, b:nat. (((((a \neq 0) \lor (b \neq 0)) \land (gcd(a, b) = gcd(old_a, old_b))) \land (\neg ((a > 0) \land (b > 0)))) \Rightarrow (let result = if a = 0 then b else a in }
\end{verbatim}
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![Code Relevant to Verification Condition “Is Result Correct?”](image)

Furthermore, by selecting the condition with a single click, the editor displays the view shown in Figure 4. The postcondition to be proved is underlined in green while grey underlines indicate those parts of the program that contributed to the derivation of the weakest precondition (in case of a loop, only the loop expression is underlined; actually, most of the verification condition comes from the invariant of the loop). It should be noted that the body of the loop does here play no role, because for the derivation of the weakest precondition only the invariant of the loop is considered. This condition (like all the other verification conditions) can be automatically checked by a double mouse click.

### 3.2 Visualization of Procedure Execution and Formula Evaluation

Since version 2.1.0 RISCAL provides both the visualization of execution traces [23] and of formula evaluation [24]. For this the graphical user interface depicts a row “Visualization” with two (mutually exclusive) check box options “Trace” and “Tree”.

By selecting the option “Trace” the execution of a procedure displays the changes of the variable values by the commands in the body of a procedure; for functions (including predicates and theorems) whose result is determined by the evaluation of an expression the computation of the result is displayed as a single step (except that also calls of other operations are displayed). This mode of visualization is thus mainly useful for understanding the operational behavior of a procedure. For instance, take the following RISCAL model of the “Bubble sort” algorithm:

```riscal
proc gcdp(m:nat, n:nat): nat
  requires m\geq 0 \land n\geq 0;
  ensures result = gcd(m,n);
{
  var a:nat = m;
  var b:nat = n;
  while a > 0 \land b > 0 do
    invariant a \neq 0 \lor b \neq 0;
    invariant gcd(a,b) = gcd(old_a,old_b);
    decreases a+b;
    { 
      if a > b then 
        a = a\%b;
      else 
        b = b\%a;
    }
  return if a = 0 then b else a;
}
```

```riscal
proc cswap(a:array, i:index, j:index): array
{ 
  var b:array = a;
  if b[i] > b[j] then 
    var x:elem := b[i];
    b[i] := b[j];
    b[j] := x;
  }
  return b;
}
```

```riscal
proc bubbleSort(a:array): array {
```

Figure 4: Code Relevant to Verification Condition “Is Result Correct?”

(request = gcd(m, n)))

Furthermore, by selecting the condition with a single click, the editor displays the view shown in Figure 4. The postcondition to be proved is underlined in green while grey underlines indicate those parts of the program that contributed to the derivation of the weakest precondition (in case of a loop, only the loop expression is underlined; actually, most of the verification condition comes from the invariant of the loop). It should be noted that the body of the loop does here play no role, because for the derivation of the weakest precondition only the invariant of the loop is considered. This condition (like all the other verification conditions) can be automatically checked by a double mouse click.

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```riscal
val N:nat; val M:nat;
type index = Z[-N,N]; type elem = Z[-M,M]; type array = Array[N, elem];

proc cswap(a:array, i:index, j:index): array 
{ 
  var b:array = a;
  if b[i] > b[j] then 
    var x:elem := b[i];
    b[i] := b[j];
    b[j] := x;
  }
  return b;
}
```

```riscal
proc bubbleSort(a:array): array {
```

Figure 4: Code Relevant to Verification Condition “Is Result Correct?”

(request = gcd(m, n)))

Furthermore, by selecting the condition with a single click, the editor displays the view shown in Figure 4. The postcondition to be proved is underlined in green while grey underlines indicate those parts of the program that contributed to the derivation of the weakest precondition (in case of a loop, only the loop expression is underlined; actually, most of the verification condition comes from the invariant of the loop). It should be noted that the body of the loop does here play no role, because for the derivation of the weakest precondition only the invariant of the loop is considered. This condition (like all the other verification conditions) can be automatically checked by a double mouse click.

```riscal
val N:nat; val M:nat;
type index = Z[-N,N]; type elem = Z[-M,M]; type array = Array[N, elem];

proc cswap(a:array, i:index, j:index): array 
{ 
  var b:array = a;
  if b[i] > b[j] then 
    var x:elem := b[i];
    b[i] := b[j];
    b[j] := x;
  }
  return b;
}
```

```riscal
proc bubbleSort(a:array): array {
```
Wolfgang Schreiner

Figure 5: Visualization of Bubble Sort

```scala
var b: array = a;
for var i:index := 0; i < N-1; i := i+1 do {
    for var j:index := 0; j < N-i-1; j := j+1 do
        b := cswap(b,j,j+1);
}
return b;
}

If we select the operation `bubbleSort` and set with the button “Other Values” the parameters \(N\) and \(M\) to 4 and 3, respectively, the second check of the procedure is performed with the argument array \(a = [-2,-3,-3,-3]\) upon which the window displayed in Figure 5 pops up. The window displays a directed graph (a linear sequence of nodes) that are connected by directed edges (arrows) and that are laid out in a two-dimensional manner from left to right and top to bottom. The numbered nodes in this graph represent the sequence of states constructed by performing assignments to the variables of the procedure; by hovering with the mouse pointer over such a node, a small window pops up that displays the values of the various variables in that state (see the node numbered 17 in the figure). The nodes with a graph symbol represents the call of another operation, the header of this window displays the call itself. By double-clicking on such a call node which represents the application of an operation the content of the window is modified to visualize the execution of that operation (moving to another level of visualization). A double click on any empty part of the window moves the display back to the previous level.

The option “Tree” triggers the visualization of the evaluation of logic formulas. As an example, we visualize in the following the evaluation of the formula introduced by the following definition of predicate `forallPexistsQFormula()`:

```scala
val N = 4;
type T = N[N];
pred p(x:T) ⇔ x < N;
```
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Figure 6: The Visualization of the Evaluation of \((\forall x. \ p(x) \Rightarrow \exists y. \ q(x,y))\)

\[
\text{pred } q(x:T,y:T) \Leftrightarrow x + 1 = y;
\]
\[
\text{pred } \text{forallPexistsQFormula()} \Leftrightarrow \forall x:T. \ p(x) \Rightarrow \exists y:T. \ q(x,y);
\]

If this formula is evaluated, the window depicted in Figure 6 pops up which displays the evaluation tree for that formula. The root of that tree represents the evaluation of the predicate for all possible arguments. Numbered nodes are children of the root node each of which represents the evaluation of the predicate for one particular argument. Labeled nodes represent propositional formulas or quantified formulas where the label denotes the formula’s outermost logical symbol (logical connective or quantifier). By hovering with the mouse pointer over such a node, a small window pops up that displays the formula, the values of its free variables (actually the values of all variables visible at the occurrence of the formula), and the truth value of the formula. Atomic formulas with application of user-defined predicates are displayed by a node with a graph symbol. By double-clicking on such a node, the visualization switches from the current formula to the formula defining the predicate.

Every evaluation tree is pruned such that it only displays the information necessary to understand how its truth value was derived. For instance, if the truth value of a conjunction \(F_1 \land F_2\) is “false”, only the first subformula is displayed whose truth value is “false”. Likewise, If the truth value of an existentially quantified formula \(\exists x. \ F\) is “true”, only one instance \(F[x \mapsto a]\) is displayed, where \(a\) is the first value encountered in the evaluation that makes \(F\) “true”. By inspecting such an evaluation tree, we may quickly get the essential information to understand how the (possibly unexpected) truth value of a formula (e.g., of an invalid verification condition) was derived.

The scalability of the RISCAL visualization tools is currently limited: if the visualized graph (execution trace or evaluation tree) exceeds the actual area of the window, only a part of the graph is displayed and scrollbars are shown that allow the user to navigate the displayed part within the graph; this is be-
cause the Eclipse GEF4 Zest software underlying the RISCAL visualization does not support zooming. There are also limits on the size of the evaluation tree for which the algorithms used by this software can compute a layout in a reasonable amount of time; thus RISCAL currently only visualizes trees with a configurable maximum (currently: 500) of nodes on each layer of abstraction.

3.3 Web-Based Exercises

As will be discussed in the following sections, the use of RISCAL in educational scenarios has so far required the installation of software on every student’s own computer, either a virtualization software (for the execution of a downloadable virtual machine that has RISCAL preinstalled) or a remote desktop solution (for the execution of RISCAL on a remote server). However, especially in undergraduate education it would be advisable to provide access to RISCAL also via the web without requiring any local software installation. Furthermore, the elaboration of an exercise should not necessarily expose a complete RISCAL specification but only those parts that are relevant for the student (hiding in particular those parts that describe how the correctness of the exercise is checked). For this purpose, we have recently developed the “RISCAL WebEx” software [21] (not to be confused with the CISCO WebEx video conferencing software), an experimental Python-based software framework that allows to turn with moderate effort annotated RISCAL specifications into web-based exercises.

Figure 7 displays the architecture of this software and the workflow of its use. In a nutshell, the command `webex` generates from an annotated RISCAL specification file `exercise.txt` (that is deployed on the execution server) an HTML file `exercise.html` that represents an exercise form; this file is deployed on the web server. The client downloads `exercise.html` from the web server to her web browser. She enters her name into the exercise form and performs some task described in the form by providing some input and then triggering some RISCAL action to check that input. The execution service `webex.wsgi` runs
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Figure 8: A RISCAL Web Exercise

on the execution server, a network server that directly communicates with the user’s web browser. This execution service produces from exercise.txt and the user input a plain RISCAL file and starts a RISCAL process to execute the action denoted by the user on that file. When the RISCAL process terminates (normally, by user abortion, or by timeout), webex.wsgi returns the result status of the RISCAL process (success or failure) together with the produced output and a digital certificate of the execution (signed with a private key of the execution server) to the web browser; the certificate includes the user name, the action performed, and the grade points earned. Ultimately, from the certificates of all tasks that the user has performed, a summary certificate is produced that the user may store in a file and submit as the result of the exercise. The lecturer may use a script webex.verify (which needs access to the private server key) to check the authenticity of the certificate and to retrieve the number of grade points that she has earned. Furthermore, an existing exercise file can be instantiated by a script webex.template with data from another file, thus producing a collection of exercises.

Figure 8 displays an example of such a web exercise generated from an annotated RISCAL file whose content is sketched below:

```riscal
//@ <webex id="firstOrderPragmatics" type="riscal" header="Exercise: Formal Specification">
//@ <display>

TASK DESCRIPTION:
In the following we consider arrays of maximum length \(N\) whose elements are natural numbers of maximum size \(M\):

val \(N\) = 4 ; // choose small values
val \(M\) = 3 ;
type Elem = \(\mathbb{N}(M)\); type Arr = Array[N,Elem]; type Index = \(\mathbb{Z}(-1,N)\);

Take the problem of finding the smallest index \(i\) at which an element \(e\) occurs among the first \(n\) elements of an array \(a\). Your task is to develop a formal specification of this problem, i.e., to define a predicate \(P(a,n,e)\), the input condition of the problem, and a predicate \(Q(a,n,e,i)\), the output condition.

pred \(P(a,n,:Index, e:Elem)\) =

// Formulate here the input condition
\(0 \leq i < n \land a[i] = e\)

pred \(Q(a,n,:Index, :Index)\) =

// Formulate here the output condition
\(0 \leq i \land n \land a[i] = e \land i\)


TASK: check whether your specification adequately specifies the following code ...

Check correct() Execution completed for ALL inputs (268 ms, 1296 checked, 2808 inadmissible). SUCCESS termination.

```
In the following we consider arrays of maximum length $N$ whose elements are natural numbers of maximum size $M$:

```plaintext
//@ <public>
val N = /*@<input id="N" cols="2">*/4/*@</input>@*/; // choose small values
val M = /*@<input id="M" cols="2">*/3/*@</input>@*/;
type Elem = N[M]; type Arr = Array[N,Elem]; type Index = Z[-1,N];
//@ <private>
...
/*@ <button id="correct" label="Check correct()" action="correct">
<argument value="-opt-si"> <argument value="0">
</button>*/
```

This exercise combines displayed text with formal content (some of which is hidden from the student) and input from the student. After having provided the requested input, the student presses the button upon which the RISCAL software on the remote server is invoked to check its correctness; the success status and the output of the check (for further investigation in case of errors) are displayed by a success icon respectively in a (resizable) output field.

The RISCAL WebEx software is still in an experimental stage; more details on its use (including the exact grammar of the annotation language) are given in its manual [21]; the RISCAL web page [14] lists more examples of RISCAL-based web exercises.

4 The Application of RISCAL in Education

In this section we are going to report on applications of RISCAL in educational scenarios, past, current, and future (respectively envisioned) ones.

4.1 A Course on Formal Methods

The main use of RISCAL so far has been in an annual course on “Formal Methods in Software Development” at the Johannes Kepler University (JKU) Linz for master students in computer science and computer mathematics. This course deals with the formal specification and verification (respectively falsification) of computer programs and is organized in three parts: the first part deals with fundamental concepts and calculi (the Hoare calculus, Dijkstra’s predicate transformer calculus, a relational calculus), the second part with the application of these techniques to actual Java programs, the third one with concurrency and nondeterminism. All parts are accompanied by the demonstration of software tools and their use by students to elaborate ten home assignments. The student groups are of moderate size (about 25 participants) and have adequate technical background to install and operate on their own computers a virtual machine with RISCAL preinstalled.

Before the academic year 2017/2018, we used in the first part of this course (with four home assignments) the RISC ProofNavigator [17] as an interactive proving assistant on top of which the RISC ProgramExplorer [18] provided an integrated specification and verification environment for a subset of Java called “MiniJava”. With these tools, the adequacy of specifications and the correctness of programs with respect to these specifications could in essence only be judged by proving the validity of (manually derived respectively automatically generated) verification conditions; if an attempt to such a proof failed, it remained often unclear to students whether this was due to a poor proof strategy or inadequate formal specifications respectively annotations (to at least rule out actual program errors, correct programs were
handed out). Furthermore, sometimes a verification succeeded only because the student’s specification was inadequate (e.g., a postcondition was trivially satisfied due to an error in the logical formulation).

In the winter semester 2017/2018, we complemented the first four assignments by the use of RISCAL version 1.x. The topic of Assignment 1 was the development of formal specifications and the analysis of their adequacy in various ways [22]:

- by executing functions that were implicitly defined from the pre- and postconditions (illustrating the input/output relationship expressed by the specification);
- by checking whether preconditions and postconditions are satisfiable but non-trivial (ruling out logical errors that make the specification trivial);
- by checking whether postconditions define the results uniquely (if intended);
- by checking whether every output of a given procedure that implements the specification indeed satisfies the output condition.

For these checks students had to formulate the respective functions and theorems manually. Assignment 2 dealt with formal verification and the corresponding annotation with loops by invariants; students first had to validate specifications and invariants (by checking their validity in all possible executions of RISCAL procedures) and then to manually derive the verification conditions and check their validity with RISCAL (which implies that the invariants are sufficiently strong). In Assignments 3 and 4, specifications and invariants were first validated as in Assignment 2 (without generating actual verification conditions) and then transferred to a corresponding MiniJava program; the verifications conditions were generated by the RISC ProgramExplorer and interactively verified with the RISC ProofNavigator. The use of RISCAL in the later assignments ensured that proofs were only attempted after validation had ensured that specifications and annotations were not too strong (but invariants might be still too weak).

In the winter semester 2018/2019, RISCAL version 2.x enabled us to augment the assignments: First, in all assignments the visualization of formula evaluation trees could be used to better understand how the truth values of formulas were derived; furthermore, in Assignments 2–4 the conditions for specification validation conditions and for program verification were automatically generated from the annotated programs, such that in Assignments 3 and 4, the specifications and annotations could be analyzed more thoroughly than in the previous year.

A final anonymous evaluation of the course software by the participants (13 reports were returned from a total of 21 students graded) indeed indicated a very high satisfaction with the use of RISCAL both from the point of ease of use (average rating: 4.2 of 5) and learning success (average rating: 4.3 of 5); the ratings were significantly better than for our previously developed RISC ProgramExplorer/ProofNavigator (ease of use: 3.5, learning success: 3.5) and summarily higher than for any other of the six toolsets used in the course (the second most popular one was ESC/Java2 with ratings of 4.2 and 3.8, respectively). As for the actual effect on exercise grading, however, we have to report a null result: as in the previous two incarnations of the course the average grades were in the mostly range of 80–95 points out of 100 with no statistically significant effect visible by the use of RISCAL.

Furthermore, a recently completed bachelor thesis [13] has elaborated the formal specification and verification of fundamental searching and sorting algorithms (including the major asymptotically fast ones) on sequences in various representations (arrays, recursive lists, pointer-linked lists); here the student was able to elaborate on his own annotations (loop invariants) that are sufficiently strong to generate verification conditions that pass all RISCAL checks in finite models. This is to us a strong indication that the use of RISCAL may be indeed able to quickly enhance a student’s understanding of the formalization of theories and algorithms.
4.2 A Course on Logic

Since 2013 we have collaborated with three colleagues in teaching JKU’s introductory “Logic” course for (about 200) first semester undergraduate students in computer science. This course consists of three modules on propositional logic/satisfiability solving (SAT), first order predicate logic (FO), and satisfiability module theories (SMT). Having started with a traditional “paper and pencil” course, we have over the years incorporated the use of various logic-based software tools: a SAT solver (Limboole), an interactive proving assistant for predicate logic (the RISC ProofNavigator), an automated reasoning system (Theorema), and SMT solvers (Boolector, Z3). However, we have refrained from making the use of these tools mandatory, because they must be installed on the students’ own computers and we could not provide sufficient technical support for a large number of course participants (all beginners with varying technical background), if they were obliged to use the software. Until the academic year 2017/2018, we therefore confined the use of these tools to four voluntary “laboratory assignments” in which students had the possibility to earn additional grade points by solving specific problems with these tools. Since these assignments were outside the main stream of the course, only a minor fraction of the students (about 5–10%) took this opportunity, either because of special interest in the topic or just because of an urgent need for extra grade points to pass the course. While we thus limited our support efforts, we also artificially constrained the potential educational effects of the use of logic software.

However, in the current winter semester 2018/2019, we have experimentally integrated into the course an additional element to foster the more wide-spread use of these tools (still without making it compulsory): every week we give voluntary “bonus assignments” (twelve in total) by which a student may earn up to 20% of the grade points that are awarded by the weekly written tests on which the overall grading of the course is mainly based. These assignments are of a moderate complexity (much smaller than that of the laboratory assignments), are to be performed by the students at home, and are to be submitted before the corresponding tests; furthermore the assignments are designed such that the correctness of results can be essentially self-checked by the students via the utilized software. Since thus mostly correct solutions are submitted and since also only a minor part of the grading is based on these home assignments (typically sufficient to improve the grade by one level), essentially only the completeness and plausibility of solutions is judged by teaching assistants. To mitigate the effect of cheating, we include some element of “personalization” to the assignment which ranges from the automatized generation of individualized problem inputs for every student to simply requiring from every student to submit a screenshot that demonstrates her personal use of the software.

In this ongoing effort, RISCAL is being used for one of the laboratory assignments (replacing the previous use of the RISC ProofNavigator) and for three of the bonus assignments. All these assignments are associated to the first half of the module on first order predicate logic dealing with its syntax, semantics, and pragmatics (the other half of this module mainly deals with proving for which the Theorema system is used). In the current semester, the use of RISCAL is still based on a virtual machine installation and on a remote server installation (the first prototype of the web-based framework for RISCAL exercises described in Section 3.3 has been just recently completed and will be used from the next academic year on). Thus each RISCAL assignment currently consists of one or more RISCAL skeleton specifications where the students have to complete some definitions and then execute some operations to check the correctness of their solutions:

- In the “syntax” assignment, students have to parse given predicate logic formulas and include parentheses to make their structure unique; it is the logic equivalence to the original formula that is checked. Furthermore, students have to translate informal statements to logic formulas; here the logic equivalence to another formalization is checked.
• In the “semantics” assignment, students have to determine the satisfying assignments of given formulas (respectively determine the truth values of closed formulas); operations are given that not only check the correctness of the solutions but also give hints (such as the number of expected solutions or the truth values arising from all possible assignments). Furthermore, students have to decide whether one formula is a logical consequence of (respectively logically equivalent to) another one and to transform a formula into a logical equivalent form with a certain syntactic property; again the results can be checked (at least validated) by executing some given operations.

• In the “pragmatics” assignment, students have to translate several given informal problem specifications into logic formulas that capture the input respectively output conditions of the problems; this partially also involves the definition of auxiliary functions and predicates. The results can be validated by executing some operations as described in Section 4.2.

As the results of the RISCAL assignments, students submit the completed specification file (together with a screenshot demonstrating the personal use of the software).

Concerning the actual effects on the use of RISCAL and other software used in this course we can report both on an anonymous student evaluation (we received evaluations from 133 students) and the actual outcomes of the tests (197 students were graded, 24 students dropped out after the first or second lecture without receiving a grade):

• The question “Which software was helpful?” was answered by 82 students with “RISCAL”, clearly trailing “Limboole” (named 121 times) but significantly ahead of the other two tools (which were named 51 and 54 times); so among those tools applicable to richer logics than pure propositional logic, RISCAL was the most popular one. Also there was only a very small number of problems reported with the virtual machine (5), documentation (3), and ease of use (6).

• As for the question “Why did you attempt the bonus assignments?” (no classification with respect to the specific tool), only a disappointing number of 37 answered by “interest” or the potential of better “understanding”, while 77 reported as their core reason “easiness” or the potential to earn additional “points”. Furthermore, the question “Would you like the introduced software to be part of your other courses in the curriculum?” was answered positively only by 41 students with “Limboole” mentioned 25 times and RISCAL mentioned 12 times (the other two tools were mentioned 10 and 11 times). Interestingly, however, the most positive impact on interest in the course was attributed to “Software” (named 18 times), while the most positive impact on actually understanding was attributed to “Exercises” (named 36 times) and “Moodle Quizzes” (named 26 times) much more than “Software” (named 13 times).

• As for the actual impact on grading, we report subsequently in detail only on the use of RISCAL. 85 out of 191 participating students attempted at least one of the three RISCAL-related bonus exercises (each bonus exercise was performed by 76, 74, and 54 students, respectively). Since these are many more students than those who did a voluntary RISCAL-related laboratory exercise (9), our goal to spread the use of logic software tools among students was thus apparently achieved. The majority of the submissions indeed earned the students the complete bonus of one point per bonus assignment; since, however, the maximum of classroom exercise and bonus exercise could not exceed 5 points, in average only 1.48 extra points per student actually contributed to the grading.

• There was a strong correlation between work on bonus assignments and performance in classroom assignments: out of 85 students with all three classroom assignments positive also 56 students did some bonus assignment; however, out of 54 students with only zero or one positive classroom
assignments only 7 did some bonus assignment (which actually enabled 3 of them to pass the threshold of two positive assignments). Interestingly, also of the 9 students who did the lab exercise, 5 students had already 3 positive classroom assignments (but apparently wanted to improve their grade) while 1 student had 1 such assignment (and passed by the laboratory the threshold of 2 positive assignments). This may indicate that the performance of students improves by the use of the tool but also that only students with a higher performance use the tools (probably both are true). Thus, while a strong correlation is visible, the direction of cause and effect is not clear.

- As for the overall effect on grading, a comparison between the current invocation of the module and the corresponding instances in the previous year is difficult, since some of the assignments have considerably changed. All in all a significant difference in the overall outcome of the RISC-related part of the first-order logic module is not discernible (the overall number of about 3.4 points earned in the classroom exercises, not considering the bonus exercises, remained almost the same).

In a nutshell, our overall experience is that indeed a large number of students attempted the RISCAL bonus exercises (and performed well there) and that those who did so achieved significantly better results in the classroom exercises than those that did not. However it also seems that really weak students (subsequently failing the course) mostly ignored the software and thus also did not make use of extra chances to pass the course. Furthermore, the final evaluation indicates that the main motivation of many students to use the software was to earn additional points rather than an intrinsic interest or the desire to improve their understanding; nevertheless “Software” was listed as the highest impact factor for raising their interest (while it was not named as a significant factor for improving their understanding).

4.3 Courses on Modeling and Programming

In the forth-coming summer semester 2019, we will start at JKU (as a collaborative effort of several lecturers) a new course on “Formal Modeling” for undergraduate students in mathematics. Approximately one third of this course will deal with the modeling of computational problems respectively computational systems in first order logic; here we will apply RISCAL to check the validity of the models respectively to illustrate their executable aspects. This exposition will complement other already existing courses in the degree program that mainly focus on modeling continuous domains in science and engineering with techniques from analysis and numerical mathematics.

The audience/level of this course will differ from those of the previously described courses on “Logic” (for first semester students in computer science) and “Formal Methods in Software Development” (for master students in computer science and computer mathematics) by addressing students of mathematics that are in the later phase of their bachelor program. Likewise the content of this course differs from that on “Logic” (focusing on the basics of first order predicate logic) and “Formal Methods” (focusing on program specification and verification) by emphasizing the applicability of logic modeling to problems in discrete mathematics, computer algebra, computational logic, etc. The goal is to demonstrate also to mathematicians the applicability and practical importance of logic-based modeling techniques which allows one also to draw definite (precise and formally justifiable) conclusions about certain problem domains (rather than just computing numerically approximated solutions).

As a longer-term vision we hope to achieve some impact also on the basic education of computer science students in the actual core of their field, the design and implementation of computer programs. Corresponding introductory courses on “Software Development” or “Algorithms and Data Structures” typically present the field on the basis of examples (which students mimic/adapt/generalize to develop their own solutions to given problems) respectively of general recipes whose propriety (domain of ap-
plicability and correctness) however is again explained by examples; consequently students are trained to judge the adequacy of their own developments by example-based “trial and error” (also known as “testing”). Thus in our own experience even master students of computer science are hard-pressed when given the task to develop from a simple informal problem specification a code snippet of twenty lines that solves the problem in a completely correct way (considering also all special, boundary, respectively corner cases). It is hard to imagine how on this basis complex software systems can be developed that are expected to be reliable, safe, and secure.

A possible approach to improve this situation is to incorporate into the education on algorithm respectively software development also assignments where the correctness of the solution is automatically checked on the basis of logic-based software. For example, as sketched in Section 3.3 a RISCAL-based web assignment may

- display an informal description of a computational problem to be solved,
- provide the public skeleton of a procedure with an embedded area where the student may enter the code of their implementation to solve this problem,
- contain a hidden annotation of the code by a formal specification of its pre- and post-condition,
- expose a button to execute the procedure for all possible inputs.

The student may then self-check the correctness of her solution: if the solution fails for some input argument (i.e., the code aborts or produces a wrong result), the student is correspondingly informed from the RISCAL execution output and may correct her solution. Since solutions have been self-checked, it can be expected that mostly correct solutions will be submitted; furthermore, the results can be by this logic-based execution framework automatically graded with respect to its formal contract (extending the functionality of conventional autograder software [25] which automatically grades programs by testing [9] or static code analysis [26]).

While we are not responsible for any course that is suitable to embody the vision sketched above, we will approach some lecturers responsible for the beginners’ education in algorithm and software development at JKU; we hope to convince them by a demonstration of the capabilities of RISCAL (with appropriately prepared examples and lecturing materials) to experimentally include some of these envisioned elements into their own courses.

5 Conclusions

Compared with the status reported in [22], RISCAL has significantly matured by incorporating additional features that are relevant its application in education. Most important, however, we have started to gain actual experience with the use of the software for educational scenarios, in particular in graduate classes on formal methods and logic and in the elaboration of two bachelor theses on formal specification [5] and verification [13]. This first experience not only shows that the software is stable and (compared to other software we have used before) easy to use; it also shows that students are really able to develop formal material in a faster way and of substantially higher quality than with the proof-based tools we have used before. Actually, it is the first time we had the feeling that (some) students enjoyed using the software due to its immediate feedback on the meaning and adequacy of the formalisms they develop.

As for our initial experience with the (optional) use of RISCAL in an undergraduate introductory course on logic, we also experienced that students had little technical problems with RISCAL. Furthermore, there was a strong correlation visible between the use of the software by students and their subsequent performance in traditional classroom exercises. However, whether the software indeed improved
their understanding or whether just the stronger students used the software remains unclear; students that performed poorly in the classroom assignments generally ignored the software. A subsequent evaluation revealed that students were mainly motivated to use the software for improving their grades; nevertheless the factor “Software” was most often named as a factor to raise interest in the presented topics. This experience may indicate that the use of RISCAL-like tools in education may achieve true profit once the students already know enough to appreciate the potential of such tools; in the initial stages of their education, they may serve more as additional factors to raise subsequent interest in the presented topics.

However, all this is still mainly hypothesis based on anecdotal evidence. In addition to enhancing RISCAL further (e.g., by applying also SMT based checking techniques) and to applying RISCAL in more courses (e.g., on formal modeling mathematical domains), our main next goal is to evaluate the educational effect of the use of this tool in a systematic way and provide statistically sound scientific evidence of its effects. The core difficulty will be to design a suitable experimental scenario and integrate it into courses such that on the one side actual scientific evidence is gathered (we have to avoid, e.g., self selection bias) and on the other hand the actual education is not compromised (we cannot force students into an experiment with potential detrimental effects on their educational outcome), also considering ethical problems (how can we e.g. justify to place students into a control group that does not use the software even if we have already strong evidence that it improves their study results). Here more and deeper investigations are required.

References


Theorem and Algorithm Checking for Courses on Logic and Formal Methods


