Idempotent Generalization is Infinitary

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Abstract

Let $\mathbf{I}_S$ be an equational theory s.t. for each $f \in S$, $f(x,x) = x$. Such an equational theory is said to be *idempotent*. It is known that the anti-unification problem (AUP) $f(a,b) \triangleq g(a,b)$ modulo $\mathbf{I}_{\{f,g\}}$ admits infinitely many least-general generalizers (lggs) [1]. We show that, modulo $\mathbf{I}_{\{f\}}$, $f(a, f(a,b)) \triangleq f(b, f(a,b))$ admits infinitely many lggs.

Consider the incomparable lggs: $g_1 = f(f(x, f(x,b)), f(a, f(x,b)))$ and $g_2 = f(f(x, f(a,x)), f(f(x,b), f(a,b)))$. Note, $g_1\{x \leftarrow a\} \approx_{\mathbf{I}_{\{f\}}} g_2\{x \leftarrow a\} \approx_{\mathbf{I}_{\{f\}}} f(a, f(a,b))$ and $g_1\{x \leftarrow b\} \approx_{\mathbf{I}_{\{f\}}} g_2\{x \leftarrow b\} \approx_{\mathbf{I}_{\{f\}}} f(b, f(a,b))$. Let $S(i+1) = f(g_1, f(g_2, S(i)))$ and $S(0) = g_1$. The set of incomparable lggs $\{S(n) | n \in \mathbb{N}\}$ is infinite.

References

[1] Loïc Pottier. Generalisation de termes en theorie equationnelle. Cas associatif-commutatif. Research Report RR-1056, INRIA, 1989.