Verifying Concurrent Systems

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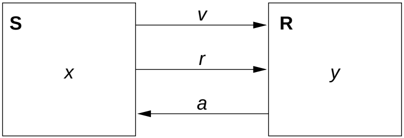
1. **Verification by Computer-Supported Proving**

2. Verification by Automatic Model Checking

A Bit Transmission Protocol



var x, y
var $v := 0, r := 0, a := 0$

S: **loop**
  choose $x \in \{0, 1\}$
  $1 : v, r := x, 1$
  $2 :$ **wait** $a = 1$
  $r := 0$
  $3 :$ **wait** $a = 0$

$||$

R: **loop**
  $1 :$ **wait** $r = 1$
  $y, a := v, 1$
  $2 :$ **wait** $r = 0$
  $a := 0$

Transmit a bit through a wire.

A (Simplified) Model of the Protocol

$State := PC^2 \times (\mathbb{N}_2)^5$

$$
\begin{aligned}
& I(p,q,x,y,v,r,a) :\Leftrightarrow p = q = 1 \wedge x \in \mathbb{N}_2 \wedge v = r = a = 0. \\
& R(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad S1(\ldots) \vee S2(\ldots) \vee S3(\ldots) \vee R1(\ldots) \vee R2(\ldots).
\end{aligned}
$$

$$
\begin{aligned}
& S1(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad p = 1 \wedge p' = 2 \wedge v' = x \wedge r' = 1 \wedge \\
& \quad q' = q \wedge x' = x \wedge y' = y \wedge v' = v \wedge a' = a. \\
& S2(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad p = 2 \wedge p' = 3 \wedge a = 1 \wedge r' = 0 \wedge \\
& \quad q' = q \wedge x' = x \wedge y' = y \wedge v' = v \wedge a' = a. \\
& S3(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad p = 3 \wedge p' = 1 \wedge a = 0 \wedge x' \in \mathbb{N}_2 \wedge \\
& \quad q' = q \wedge y' = y \wedge v' = v \wedge r' = r \wedge a' = a. \\
& R1(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad q = 1 \wedge q' = 2 \wedge r = 1 \wedge y' = v \wedge a' = 1 \wedge \\
& \quad p' = p \wedge x' = x \wedge v' = v \wedge r' = r. \\
& R2(\langle p,q,x,y,v,r,a\rangle, \langle p',q',x',y',v',r',a'\rangle) :\Leftrightarrow \\
& \quad q = 2 \wedge q' = 1 \wedge r = 0 \wedge a' = 0 \wedge \\
& \quad p' = p \wedge x' = x \wedge y' = y \wedge v' = v \wedge r' = r.
\end{aligned}
$$

A Verification Task

$$\langle I, R \rangle \models \Box(q = 2 \Rightarrow y = x)$$

$$\mathit{Invariant}(p, \ldots) \Rightarrow (q = 2 \Rightarrow y = x)$$

$$I(p, \ldots) \Rightarrow \mathit{Invariant}(p, \ldots)$$
$$R(\langle p, \ldots \rangle, \langle p', \ldots \rangle) \land \mathit{Invariant}(p, \ldots) \Rightarrow \mathit{Invariant}(p', \ldots)$$

$$\begin{array}{l}
\mathit{Invariant}(p, q, x, y, v, r, a) :\Leftrightarrow \\
\quad (p = 1 \lor p = 2 \lor p = 3) \land (q = 1 \lor q = 2) \land \\
\quad (x = 0 \lor x = 1) \land (v = 0 \lor v = 1) \land (r = 0 \lor r = 1) \land (a = 0 \lor a = 1) \land \\
\quad (p = 1 \Rightarrow q = 1 \land r = 0 \land a = 0) \land \\
\quad (p = 2 \Rightarrow r = 1) \land \\
\quad (p = 3 \Rightarrow r = 0) \land \\
\quad (q = 1 \Rightarrow a = 0) \land \\
\quad (q = 2 \Rightarrow (p = 2 \lor p = 3) \land a = 1 \land y = x) \land \\
\quad (r = 1 \Rightarrow p = 2 \land v = x)
\end{array}$$

The invariant captures the essence of the protocol.

The Verification Task in PVS

```
protocol: THEORY
BEGIN
  p, q, x, y, v, r, a: nat
  p0, q0, x0, y0, v0, r0, a0: nat

  S1: bool =
    p = 1 AND p0 = 2 AND v0 = x AND r0 = 1 AND
    q0 = q AND x0 = x AND y0 = y AND v0 = v AND a0 = a
  S2: bool =
    p = 2 AND p0 = 3 AND a = 1 AND r0 = 0 AND
    q0 = q AND x0 = x AND y0 = y AND v0 = v AND a0 = a
  S3: bool =
    p = 3 AND p0 = 1 AND a = 0 AND (x0 = 0 OR x0 = 1) AND
    q0 = q AND y0 = y AND v0 = v AND r0 = r AND a0 = a
  R1: bool =
    q = 1 AND q0 = 2 AND r = 1 AND y0 = v AND a0 = 1 AND
    p0 = p AND x0 = x AND v0 = v AND r0 = r
  R2: bool =
    q = 2 AND q0 = 1 AND r = 0 AND a0 = 0 AND
    p0 = p AND x0 = x AND y0 = y AND v0 = v AND r0 = r
```

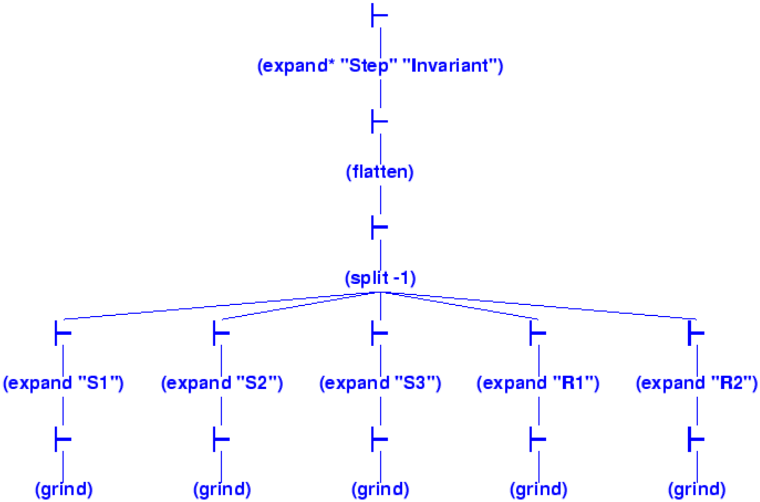
The Verification Task in PVS (Contd)

```
Init: bool =
  p = 1 AND q = 1 AND (x = 0 OR x = 1) AND
  v = 0 AND r = 0 AND a = 0
Step: bool =
  S1 OR S2 OR S3 OR R1 OR R2

Property: bool =
  q = 2 => y = x

Invariant(p, q, x, y, v, r, a: nat): bool =
   (p = 1 OR p = 2 OR p = 3) AND
   (q = 1 OR q = 2) AND
   (x = 0 OR x = 1) AND
   (v = 0 OR v = 1) AND
   (r = 0 OR r = 1) AND
   (a = 0 OR a = 1) AND
   (p = 1 => q = 1 AND r = 0 AND a = 0) AND
   (p = 2 => r = 1) AND
   (p = 3 => r = 0) AND
   (q = 1 => a = 0) AND
   (q = 2 => (p = 2 OR p = 3) AND a = 1 AND y = x) AND
   (r = 1 => (p = 2 AND v = x))
```

The Verification Task in PVS (Contd'2)

```
  VC0: THEOREM
    Invariant(p, q, x, y, v, r, a) => Property

  VC1: THEOREM
    Init => Invariant(p, q, x, y, v, r, a)

  VC2: THEOREM
    Step AND Invariant(p, q, x, y, v, r, a) =>
      Invariant(p0, q0, x0, y0, v0, r0, a0)

END protocol
```

The Proof in PVS



A Client/Server System

Client system $C_i = \langle IC_i, RC_i \rangle$.

$State := PC \times \mathbb{N}_2 \times \mathbb{N}_2$.
$Int := \{R_i, S_i, C_i\}$.

$$
\begin{aligned}
& IC_i(pc, request, answer) :\Leftrightarrow \\
& \quad pc = R \wedge request = 0 \wedge answer = 0. \\
& RC_i(I, \langle pc, request, answer \rangle, \\
& \qquad \langle pc', request', answer' \rangle) :\Leftrightarrow \\
& \quad (I = R_i \wedge pc = R \wedge request = 0 \wedge \\
& \qquad pc' = S \wedge request' = 1 \wedge answer' = answer) \vee \\
& \quad (I = S_i \wedge pc = S \wedge answer \neq 0 \wedge \\
& \qquad pc' = C \wedge request' = request \wedge answer' = 0) \vee \\
& \quad (I = C_i \wedge pc = C \wedge request = 0 \wedge \\
& \qquad pc' = R \wedge request' = 1 \wedge answer' = answer) \vee
\end{aligned}
$$

$$
\begin{aligned}
& \quad (I = \overline{REQ_i} \wedge request \neq 0 \wedge \\
& \qquad pc' = pc \wedge request' = 0 \wedge answer' = answer) \vee \\
& \quad (I = ANS_i \wedge \\
& \qquad pc' = pc \wedge request' = request \wedge answer' = 1).
\end{aligned}
$$

```
Client(ident):
  param ident
begin
  loop
    ...
 R: sendRequest()
 S: receiveAnswer()
 C: // critical region
    ...
    sendRequest()
  endloop
end Client
```

A Client/Server System (Contd)

Server system $S = \langle IS, RS \rangle$.

$State := (\mathbb{N}_3)^3 \times (\{1,2\} \rightarrow \mathbb{N}_2)^2$.
$Int := \{D1, D2, F, A1, A2, W\}$.

$$\begin{aligned}
&IS(given, waiting, sender, rbuffer, sbuffer) :\Leftrightarrow\\
&\quad given = waiting = sender = 0 \wedge\\
&\quad rbuffer(1) = rbuffer(2) = sbuffer(1) = sbuffer(2) = 0.
\end{aligned}$$

$$\begin{aligned}
&RS(I, \langle given, waiting, sender, rbuffer, sbuffer\rangle,\\
&\qquad \langle given', waiting', sender', rbuffer', sbuffer'\rangle) :\Leftrightarrow\\
&\quad \exists i \in \{1,2\} :\\
&\qquad (I = D_i \wedge sender = 0 \wedge rbuffer(i) \neq 0 \wedge\\
&\qquad sender' = i \wedge rbuffer'(i) = 0 \wedge\\
&\qquad U(given, waiting, sbuffer) \wedge\\
&\qquad \forall j \in \{1,2\} \backslash \{i\} : U_j(rbuffer)) \vee\\
&\quad \ldots
\end{aligned}$$

$U(x_1, \ldots, x_n) :\Leftrightarrow x_1' = x_1 \wedge \ldots \wedge x_n' = x_n$.
$U_j(x_1, \ldots, x_n) :\Leftrightarrow x_1'(j) = x_1(j) \wedge \ldots \wedge x_n'(j) = x_n(j)$.

```
Server:
  local given, waiting, sender
begin
  given := 0; waiting := 0
  loop
D:   sender := receiveRequest()
     if sender = given then
       if waiting = 0 then
F:       given := 0
       else
A1:      given := waiting;
         waiting := 0
         sendAnswer(given)
       endif
     elsif given = 0 then
A2:    given := sender
       sendAnswer(given)
     else
W:     waiting := sender
     endif
  endloop
end Server
```

A Client/Server System (Contd'2)

$$
\begin{aligned}
&\ldots\\
&(I = F \wedge sender \neq 0 \wedge sender = given \wedge waiting = 0 \wedge\\
&\quad given' = 0 \wedge sender' = 0 \wedge\\
&\quad U(waiting, rbuffer, sbuffer)) \vee\\
&\\
&(I = A1 \wedge sender \neq 0 \wedge sbuffer(waiting) = 0 \wedge\\
&\quad sender = given \wedge waiting \neq 0 \wedge\\
&\quad given' = waiting \wedge waiting' = 0 \wedge\\
&\quad sbuffer'(waiting) = 1 \wedge sender' = 0 \wedge\\
&\quad U(rbuffer) \wedge\\
&\quad \forall j \in \{1,2\} \backslash \{waiting\} : U_j(sbuffer)) \vee\\
&\\
&(I = A2 \wedge sender \neq 0 \wedge sbuffer(sender) = 0 \wedge\\
&\quad sender \neq given \wedge given = 0 \wedge\\
&\quad given' = sender \wedge\\
&\quad sbuffer'(sender) = 1 \wedge sender' = 0 \wedge\\
&\quad U(waiting, rbuffer) \wedge\\
&\quad \forall j \in \{1,2\} \backslash \{sender\} : U_j(sbuffer)) \vee\\
&\ldots
\end{aligned}
$$

```
Server:
  local given, waiting, sender
begin
  given := 0; waiting := 0
  loop
D:  sender := receiveRequest()
    if sender = given then
      if waiting = 0 then
F:      given := 0
      else
A1:     given := waiting;
        waiting := 0
        sendAnswer(given)
      endif
    elsif given = 0 then
A2:   given := sender
      sendAnswer(given)
    else
W:    waiting := sender
    endif
  endloop
end Server
```

A Client/Server System (Contd'3)

$$
\begin{array}{l}
\ldots \\
(l = W \land sender \neq 0 \land sender \neq given \land given \neq 0 \land \\
\quad waiting' := sender \land sender' = 0 \land \\
\quad U(given, rbuffer, sbuffer)) \lor \\
\hline
\exists i \in \{1,2\}: \\
\quad (l = REQ_i \land rbuffer'(i) = 1 \land \\
\quad\quad U(given, waiting, sender, sbuffer) \land \\
\quad\quad \forall j \in \{1,2\} \backslash \{i\} : U_j(rbuffer)) \lor \\
\quad (l = \overline{ANS_i} \land sbuffer(i) \neq 0 \land \\
\quad\quad sbuffer'(i) = 0 \land \\
\quad\quad U(given, waiting, sender, rbuffer) \land \\
\quad\quad \forall j \in \{1,2\} \backslash \{i\} : U_j(sbuffer)).
\end{array}
$$

```
Server:
  local given, waiting, sender
begin
  given := 0; waiting := 0
  loop
D:   sender := receiveRequest()
     if sender = given then
       if waiting = 0 then
F:       given := 0
       else
A1:      given := waiting;
         waiting := 0
         sendAnswer(given)
       endif
     elsif given = 0 then
A2:    given := sender
       sendAnswer(given)
     else
W:     waiting := sender
     endif
  endloop
end Server
```

A Client/Server System (Contd'4)

$$State := (\{1,2\} \to PC) \times (\{1,2\} \to \mathbb{N}_2)^2 \times (\mathbb{N}_3)^2 \times (\{1,2\} \to \mathbb{N}_2)^2$$

$$\begin{array}{l}
I(pc, request, answer, given, waiting, sender, rbuffer, sbuffer) :\Leftrightarrow \\
\quad \forall i \in \{1,2\} : IC(pc_i, request_i, answer_i) \wedge \\
\quad IS(given, waiting, sender, rbuffer, sbuffer)
\end{array}$$

$$\begin{array}{l}
R(\langle pc, request, answer, given, waiting, sender, rbuffer, sbuffer\rangle, \\
\quad \langle pc', request', answer', given', waiting', sender', rbuffer', sbuffer'\rangle) :\Leftrightarrow \\
\quad (\exists i \in \{1,2\} : RC_{local}(\langle pc_i, request_i, answer_i\rangle, \langle pc'_i, request'_i, answer'_i\rangle) \wedge \\
\qquad \langle given, waiting, sender, rbuffer, sbuffer\rangle = \\
\qquad\quad \langle given', waiting', sender', rbuffer', sbuffer'\rangle) \vee \\
\quad (RS_{local}(\langle given, waiting, sender, rbuffer, sbuffer\rangle, \\
\qquad\qquad \langle given', waiting', sender', rbuffer', sbuffer'\rangle) \wedge \\
\qquad \forall i \in \{1,2\} : \langle pc_i, request_i, answer_i\rangle = \langle pc'_i, request'_i, answer'_i\rangle) \vee \\
\quad (\exists i \in \{1,2\} : External(i, \langle request_i, answer_i, rbuffer, sbuffer\rangle, \\
\qquad\qquad\qquad\qquad \langle request'_i, answer'_i, rbuffer', sbuffer'\rangle) \wedge \\
\qquad pc = pc' \wedge \langle sender, waiting, given\rangle = \langle sender', waiting', given'\rangle)
\end{array}$$

The Verification Task

$$\langle I, R \rangle \models \Box\neg(pc_1 = C \land pc_2 = C)$$

$$
\begin{array}{l}
\mathit{Invariant}(pc, \mathit{request}, \mathit{answer}, \mathit{sender}, \mathit{given}, \mathit{waiting}, \mathit{rbuffer}, \mathit{sbuffer}) :\Leftrightarrow \\
\quad \forall i \in \{1,2\} : \\
\qquad (pc(i) = C \lor \mathit{sbuffer}(i) = 1 \lor \mathit{answer}(i) = 1 \Rightarrow \\
\qquad\quad \mathit{given} = i \land \\
\qquad\quad \forall j : j \neq i \Rightarrow pc(j) \neq C \land \mathit{sbuffer}(j) = 0 \land \mathit{answer}(j) = 0) \land \\
\qquad (pc(i) = R \Rightarrow \\
\qquad\quad \mathit{sbuffer}(i) = 0 \land \mathit{answer}(i) = 0 \land \\
\qquad\quad (i = \mathit{given} \Leftrightarrow \mathit{request}(i) = 1 \lor \mathit{rbuffer}(i) = 1 \lor \mathit{sender} = i) \land \\
\qquad\quad (\mathit{request}(i) = 0 \lor \mathit{rbuffer}(i) = 0)) \land \\
\qquad (pc(i) = S \Rightarrow \\
\qquad\quad (\mathit{sbuffer}(i) = 1 \lor \mathit{answer}(i) = 1 \Rightarrow \\
\qquad\qquad \mathit{request}(i) = 0 \land \mathit{rbuffer}(i) = 0 \land \mathit{sender} \neq i) \land \\
\qquad\quad (i \neq \mathit{given} \Rightarrow \\
\qquad\qquad \mathit{request}(i) = 0 \lor \mathit{rbuffer}(i) = 0)) \land \\
\qquad (pc(i) = C \Rightarrow \\
\qquad\quad \mathit{request}(i) = 0 \land \mathit{rbuffer}(i) = 0 \land \mathit{sender} \neq i \land \\
\qquad\quad \mathit{sbuffer}(i) = 0 \land \mathit{answer}(i) = 0) \land \\
\qquad \ldots
\end{array}
$$

The Verification Task (Contd)

$$
\begin{array}{l}
\ldots \\
(sender = 0 \wedge (request(i) = 1 \vee rbuffer(i) = 1) \Rightarrow \\
\quad sbuffer(i) = 0 \wedge answer(i) = 0) \wedge \\
(sender = i \Rightarrow \\
\quad (waiting \neq i) \wedge \\
\quad (sender = given \wedge pc(i) = R \Rightarrow \\
\quad\quad request(i) = 0 \wedge rbuffer(i) = 0) \wedge \\
\quad (pc(i) = S \wedge i \neq given \Rightarrow \\
\quad\quad request(i) = 0 \wedge rbuffer(i) = 0) \wedge \\
\quad (pc(i) = S \wedge i = given \Rightarrow \\
\quad\quad request(i) = 0 \vee rbuffer(i) = 0)) \wedge \\
(waiting = i \Rightarrow \\
\quad given \neq i \wedge pc_i = S \wedge request_i = 0 \wedge rbuffer(i) = 0 \wedge \\
\quad sbuffer_i = 0 \wedge answer(i) = 0) \wedge \\
(sbuffer(i) = 1 \Rightarrow \\
\quad answer(i) = 0 \wedge request(i) = 0 \wedge rbuffer(i) = 0)
\end{array}
$$

As usual, the invariant has been elaborated in the course of the proof.

The Verification Task in PVS

```
clientServer: THEORY
BEGIN

  % client indices and program counter constants
  Index : TYPE+ = { x: nat | x = 1 OR x = 2 } CONTAINING 1
  Index0: TYPE+ = { x: nat | x < 3 } CONTAINING 0
  PC: TYPE+ = { R, S, C }

  % client states
  pc, pc0: [ Index -> PC ]
  request, request0: [ Index -> bool ]
  answer, answer0: [ Index -> bool ]

  % server states
  given, given0: Index0
  waiting, waiting0: Index0
  sender, sender0: Index0
  rbuffer, rbuffer0: [ Index -> bool ]
  sbuffer, sbuffer0: [ Index -> bool ]
```

The Verification Task in PVS (Contd)

```
i, j: VAR Index

% ------------------------------------------------------------------------
% initial state condition
% ------------------------------------------------------------------------

IC(pc: PC, request: bool, answer: bool): bool =
  pc = R AND request = FALSE AND answer = FALSE

IS(given: Index0, waiting: Index0, sender: Index0,
   rbuffer: [ Index -> bool ], sbuffer: [ Index -> bool ]): bool =
  given = 0 AND waiting = 0 AND sender = 0 AND
  (FORALL i: rbuffer(i) = FALSE AND sbuffer(i) = FALSE)

Initial: bool =
  (FORALL i: IC(pc(i), request(i), answer(i))) AND
  IS(given, waiting, sender, rbuffer, sbuffer)
```

The Verification Task in PVS (Contd'2)

```
% ------------------------------------------------------------------------
% transition relation
% ------------------------------------------------------------------------
RC(pc:  PC, request:  bool, answer:  bool,
   pc0: PC, request0: bool, answer0: bool): bool =
  (pc = R AND request = FALSE AND
    pc0 = S AND request0 = TRUE AND answer0 = answer) OR
  (pc = S AND answer = TRUE AND
    pc0 = C AND request0 = request AND answer0 = FALSE) OR
  (pc = C AND request = FALSE AND
    pc0 = R and request0 = TRUE AND answer0 = answer)

RS(given: Index0, waiting: Index0, sender: Index0,
     rbuffer: [ Index -> bool ], sbuffer: [ Index -> bool ],
   given0: Index0, waiting0: Index0, sender0: Index0,
     rbuffer0: [ Index -> bool ], sbuffer0: [ Index -> bool ]): bool =
  (EXISTS i:
    sender = 0 AND rbuffer(i) = TRUE AND
    sender0 = i AND rbuffer0(i) = FALSE AND
    given = given0 AND waiting = waiting0 AND sbuffer = sbuffer0 AND
    FORALL j: j /= i => rbuffer(j) = rbuffer0(j)) OR
```

The Verification Task in PVS (Contd'3)

```
(sender /= 0 AND sender = given AND waiting = 0 AND
  given0 = 0 AND sender0 = 0 AND
  waiting = waiting0 AND rbuffer = rbuffer0 AND sbuffer = sbuffer0) OR

(sender /= 0 AND
  sender = given AND waiting /= 0 AND
  sbuffer(waiting) = FALSE AND % change order for type-checking
  given0 = waiting AND waiting0 = 0 AND
  sbuffer0(waiting) = TRUE AND sender0 = 0 AND
  rbuffer = rbuffer0 AND
  (FORALL j: j /= waiting => sbuffer(j) = sbuffer0(j))) OR

(sender /= 0 AND sbuffer(sender) = FALSE AND
  sender /= given AND given = 0 AND
  given0 = sender AND
  sbuffer0(sender) = TRUE AND sender0 = 0 AND
  waiting = waiting0 AND rbuffer = rbuffer0 AND
  (FORALL j: j /= sender => sbuffer(j) = sbuffer0(j))) OR

(sender /= 0 AND sender /= given AND given /= 0 AND
  waiting0 = sender AND sender0 = 0 AND
  given = given0 AND rbuffer = rbuffer0 AND sbuffer = sbuffer0)
```

The Verification Task in PVS (Contd'4)

```
External(i: Index,
          pc:  PC, request:  bool, answer:  bool,
          pc0: PC, request0: bool, answer0: bool,
          given:  Index0,  waiting: Index0,  sender: Index0,
            rbuffer: [ Index -> bool ],  sbuffer: [ Index -> bool ],
          given0: Index0, waiting0: Index0, sender0: Index0,
            rbuffer0: [ Index -> bool ], sbuffer0: [ Index -> bool ]): bool =

   (request = TRUE AND
    pc0 = pc AND request0 = FALSE AND answer0 = answer AND
    rbuffer0(i) = TRUE AND
    given = given0 AND waiting = waiting0 AND sender = sender0 AND
    sbuffer = sbuffer0 AND
    (FORALL j: j /= i => rbuffer(j) = rbuffer0(j))) OR

   (pc0 = pc AND request0 = request AND answer0 = TRUE AND
    sbuffer(i) = TRUE AND sbuffer0(i) = FALSE AND
    given = given0 AND waiting = waiting0 AND sender = sender0 AND
    rbuffer = rbuffer0 AND
    (FORALL j: j /= i => sbuffer(j) = sbuffer0(j)))
```

The Verification Task in PVS (Contd'5)

```
Next: bool =
  ((EXISTS i: RC(pc (i), request (i), answer (i),
                 pc0(i), request0(i), answer0(i)) AND
     (FORALL j: j /= i =>
       pc(j) = pc0(j) AND request(j) = request0(j) AND
       answer(j) = answer0(j))) AND
    given = given0 AND waiting = waiting0 AND sender = sender0 AND
    rbuffer = rbuffer0 AND sbuffer = sbuffer0) OR

  (RS(given,  waiting,  sender,  rbuffer,  sbuffer,
      given0, waiting0, sender0, rbuffer0, sbuffer0) AND
   (FORALL j: pc(j) = pc0(j) AND request(j) = request0(j) AND
              answer(j) = answer0(j))) OR

  (EXISTS i:
     External(i, pc (i), request (i), answer (i),
                 pc0(i), request0(i), answer0(i),
              given,  waiting,  sender,  rbuffer,  sbuffer,
              given0, waiting0, sender0, rbuffer0, sbuffer0) AND
     (FORALL j: j /= i =>
       pc(j) = pc0(j) AND request(j) = request0(j) AND
       answer(j) = answer0(j)))
```

The Verification Task in PVS (Contd'6)

```
% ----------------------------------------------------------------------
% invariant
% ----------------------------------------------------------------------
Invariant(pc: [Index->PC], request: [Index -> bool],
            answer: [Index -> bool],
          given: Index0, waiting: Index0, sender: Index0,
            rbuffer: [Index -> bool], sbuffer: [Index->bool]): bool =
  FORALL i:
    (pc(i) = C OR sbuffer(i) = TRUE OR answer(i) = TRUE =>
      given = i AND
      FORALL j: j /= i =>
         pc(j) /= C AND
         sbuffer(j) = FALSE AND answer(j) = FALSE) AND
    (pc(i) = R =>
      sbuffer(i) = FALSE AND answer(i) = FALSE AND
      (i /= given =>
        request(i) = FALSE AND rbuffer(i) = FALSE AND sender /= i)
      (i = given =>
        request(i) = TRUE OR rbuffer(i) = TRUE OR sender = i) AND
      (request(i) = FALSE OR rbuffer(i) = FALSE)) AND
```

The Verification Task in PVS (Contd'7)

```
(pc(i) = S =>
   (sbuffer(i) = TRUE OR answer(i) = TRUE =>
      request(i) = FALSE AND rbuffer(i) = FALSE AND sender /= i) AND
   (i /= given =>
      request(i) = FALSE OR rbuffer(i) = FALSE)) AND
 (pc(i) = C =>
   request(i) = FALSE AND rbuffer(i) = FALSE AND sender /= i AND
   sbuffer(i) = FALSE AND answer(i) = FALSE) AND
 (sender = 0 AND (request(i) = TRUE OR rbuffer(i) = TRUE) =>
    sbuffer(i) = FALSE AND answer(i) = FALSE) AND
 (sender = i =>
   (sender = given AND pc(i) = R =>
     request(i) = FALSE and rbuffer(i) = FALSE) AND
   (waiting /= i) AND
   (pc(i) = S AND i /= given =>
     request(i) = FALSE AND rbuffer(i) = FALSE) AND
   (pc(i) = S AND i = given =>
     request(i) = FALSE OR rbuffer(i) = FALSE)) AND
```

The Verification Task in PVS (Contd'8)

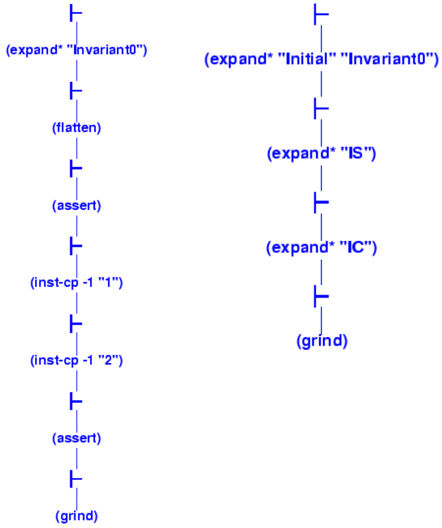
```
    (waiting = i =>
      given /= i AND
      pc(waiting) = S AND
      request(waiting) = FALSE AND rbuffer(waiting) = FALSE AND
      sbuffer(waiting) = FALSE AND answer(waiting) = FALSE) AND
    (sbuffer(i) = TRUE =>
      answer(i) = FALSE AND request(i) = FALSE AND rbuffer(i) = FALSE)

% ------------------------------------------------------------------------
% mutual exclusion proof
% ------------------------------------------------------------------------
MutEx: THEOREM
  Invariant(pc, request, answer,
    given, waiting, sender, rbuffer, sbuffer) =>
  NOT (pc(1) = C AND pc(2) = C)
```

The Verification Task in PVS (Contd'9)

```
  % -------------------------------------------------------------------------
  % invariance proof
  % -------------------------------------------------------------------------
  Inv1: THEOREM
    Initial =>
    Invariant(pc, request, answer,
      given, waiting, sender, rbuffer, sbuffer)

  Inv2: THEOREM
    Invariant(pc, request, answer,
       given, waiting, sender, rbuffer, sbuffer) AND Next =>
    Invariant(pc0, request0, answer0,
      given0, waiting0, sender0, rbuffer0, sbuffer0)

END clientServer
```

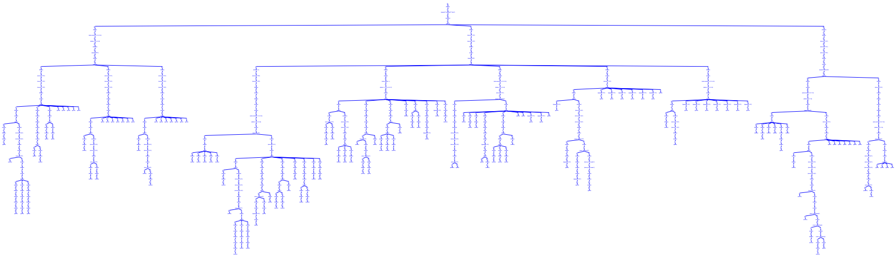
The Proof in PVS

Proofs that the system invariant implies the mutual exclusion property and that the initial condition implies the invariant.



The Proof in PVS

Proof that every system transition preserves the invariant.



- 10 subproofs, one for each transition.
  - Three from client, five from server, two from communication system.
  - Download and investigate from course Web site.

Only with computer support, verification proofs become manageable.

1. Verification by Computer-Supported Proving

2. Verification by Automatic Model Checking

The Basic Approach

Translation of the original problem to a problem in automata theory.

- Original problem: $S \models P$.
  - $S = \langle I, R \rangle$, PLTL formula P.
  - Does property P hold for every run of system S?
- Construct system automaton S_A with language $\mathcal{L}(S_A)$.
  - A language is a set of infinite words.
  - Each such word describes a system run.
  - $\mathcal{L}(S_A)$ describes the set of runs of S.
- Construct property automaton P_A with language $\mathcal{L}(P_A)$.
  - $\mathcal{L}(P_A)$ describes the set of runs satisfying P.
- Equivalent Problem: $\mathcal{L}(S_A) \subseteq \mathcal{L}(P_A)$.
  - The language of S_A must be contained in the language of P_A.

There exists an efficient algorithm to solve this problem.

Finite State Automata

A (variant of a) labeled transition system in a finite state space.

- Take finite sets *State* and *Label*.
  - The state space *State*.
  - The alphabet *Label*.
- A (finite state) automaton $A = \langle I, R, F \rangle$ over *State* and *Label*:
  - A set of initial states $I \subseteq State$.
  - A labeled transition relation $R \subseteq Label \times State \times State$.
  - A set of final states $F \subseteq State$.
    - Büchi automata: F is called the set of accepting states.

We will only consider infinite runs of Büchi automata.

Runs and Languages

- An infinite run $r = s_0 \xrightarrow{l_0} s_1 \xrightarrow{l_1} s_2 \xrightarrow{l_2} \ldots$ of automaton A:
  - $s_0 \in I$ and $R(l_i, s_i, s_{i+1})$ for all $i \in \mathbb{N}$.
  - Run r is said to read the infinite word $w(r) := \langle l_0, l_1, l_2, \ldots \rangle$.
- $A = \langle I, R, F \rangle$ accepts an infinite run r:
  - Some state $s \in F$ occurs infinitely often in r.
  - This notion of acceptance is also called Büchi acceptance.
- The language $\mathcal{L}(A)$ of automaton A:
  - $\mathcal{L}(A) := \{w(r) : A \text{ accepts } r\}$.
  - The set of words which are read by the runs accepted by A.
- Example: $\mathcal{L}(A) = (a^*bb^*a)^*a^\omega + (a^*bb^*a)^\omega = (b^*a)^\omega$.
  - $w^i = ww\ldots w$ (i occurrences of w).
  - $w^* = \{w^i : i \in \mathbb{N}\} = \{\langle\rangle, w, ww, www, \ldots\}$.
  - $w^\omega = wwww\ldots$ (infinitely often).
  - An infinite repetition of an arbitrary number of b followed by a.

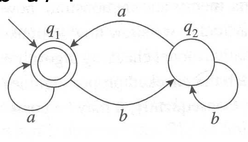


Figure 9.1
A finite automaton.

Edmund Clarke: "Model Checking", 1999.

A Finite State System as an Automaton

The automaton $S_A = \langle I, R, F \rangle$ for a finite state system $S = \langle I_S, R_S \rangle$:

- $State := States_S \cup \{\iota\}$.
  - The state space $States_S$ of S is finite; additional state ι ("iota").
- $Label := \mathbb{P}(AP)$.
  - Finite set AP of atomic propositions.

    All PLTL formulas are built from this set only.
  - Powerset $\mathbb{P}(S) := \{s : s \subseteq S\}$.
  - Every element of $Label$ is thus a set of atomic propositions.
- $I := \{\iota\}$.
  - Single initial state ι.
- $R(l, s, s') :\Leftrightarrow l = L(s') \wedge (R_S(s, s') \vee (s = \iota \wedge I_S(s')))$.
  - $L(s) := \{p \in AP : s \models p\}$.
  - Each transition is labeled by the set of atomic propositions satisfied by the successor state.
  - Thus all atomic propositions are evaluated on the successor state.
- $F := State$.
  - Every state is accepting.

A Finite State System as an Automaton

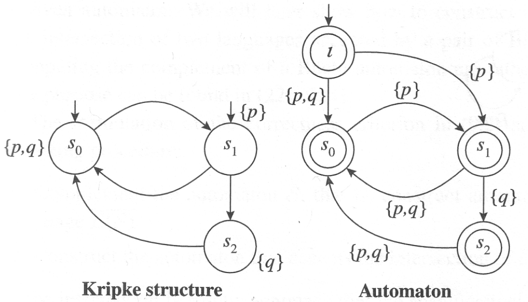


Figure 9.2
Transforming a Kripke structure into an automaton.

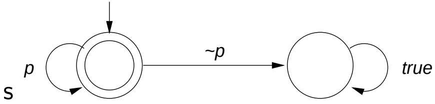
Edmund Clarke et al: "Model Checking", 1999.

If $r = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$ is a run of S, then S_A accepts the labelled version $r_l := \iota \stackrel{L(s_0)}{\rightarrow} s_0 \stackrel{L(s_1)}{\rightarrow} s_1 \stackrel{L(s_2)}{\rightarrow} s_2 \stackrel{L(s_3)}{\rightarrow} \ldots$ of r.

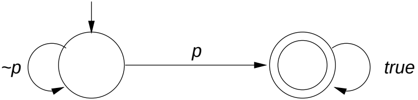
A System Property as an Automaton

Also an PLTL formula can be translated to a finite state automaton.

- We need the automaton P_A for a PLTL property P.
  - Requirement: $r \models P \Leftrightarrow P_A$ accepts r_l.
  - A run satisfies property P if and only if automaton A_P accepts the labeled version of the run.
- Example: $\Box p$.

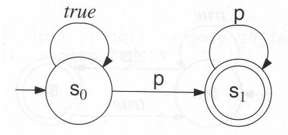


- Example: $\Diamond p$.



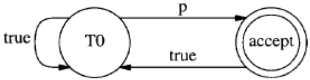
Further Examples

- Example: $\Diamond\Box p$.



Gerard Holzmann: "The Spin Model Checker", 2004.

- Example: $\Box\Diamond p$.



Gerard Holzmann: "The Model Checker Spin", 1997.

We will give later an algorithm to convert arbitrary PLTL formulas to automata.

System Properties

- State equivalence: $L(s) = L(t)$.
  - Both states have the same labels.
  - Both states satisfy the same atomic propositions in AP.
- Run equivalence: $w(r_l) = w(r'_l)$.
  - Both runs have the same sequences of labels.
  - Both runs satisfy the same PLTL formulas built over AP.
- Indistinguishability: $w(r_l) = w(r'_l) \Rightarrow (r \models P \Leftrightarrow r' \models R)$
  - PLTL formula P cannot distinguish between runs r and r' whose labeled versions read the same words.
- Consequence: $S \models P \Leftrightarrow \mathcal{L}(S_A) \subseteq \mathcal{L}(P_A)$.
  - Proof that, if every run of S satisfies P, then every word $w(r_l)$ in $\mathcal{L}(S_A)$ equals some word $w(r'_l)$ in $\mathcal{L}(P_A)$, and vice versa.
  - "Vice versa" direction relies on indistinguishability property.

The Next Steps

- Problem: $\mathcal{L}(S_A) \subseteq \mathcal{L}(P_A)$
  - Equivalent to: $\mathcal{L}(S_A) \cap \overline{\mathcal{L}(P_A)} = \emptyset$.
    - Complement $\overline{L} := \{w : w \notin L\}$.
  - Equivalent to: $\mathcal{L}(S_A) \cap \mathcal{L}(\neg P_A) = \emptyset$.
    - $\overline{\mathcal{L}(A)} = \mathcal{L}(\neg A)$.
- Equivalent Problem: $\mathcal{L}(S_A) \cap \mathcal{L}((\neg P)_A) = \emptyset$.
  - We will introduce the synchronized product automaton $A \otimes B$.
    - A transition of $A \otimes B$ represents a simultaneous transition of A and B.
  - Property: $\mathcal{L}(A) \cap \mathcal{L}(B) = \mathcal{L}(A \otimes B)$.
- Final Problem: $\mathcal{L}(S_A \otimes (\neg P)_A) = \emptyset$.
  - We have to check whether the language of this automaton is empty.
  - We have to look for a word w accepted by this automaton.
    - If no such w exists, then $S \models P$.
    - If such a $w = w(r_l)$ exists, then r is a counterexample, i.e. a run of S such that $r \not\models P$.

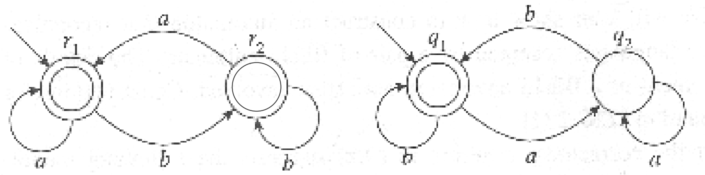
Synchronized Product of Two Automata

Given two finite automata $A = \langle I_A, R_A, State_A \rangle$ and $B = \langle I_B, R_B, F_B \rangle$.

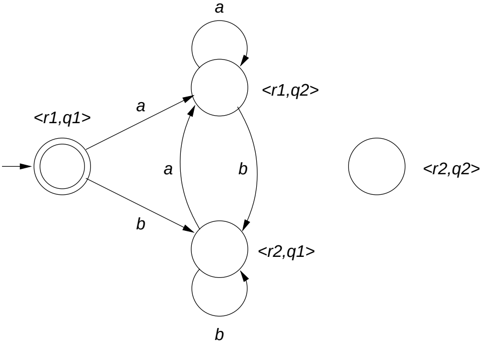
- Synchronized product $A \otimes B = \langle I, R, F \rangle$.
  - $State := State_A \times State_B$.
  - $Label := Label_A = Label_B$.
  - $I := I_A \times I_B$.
  - $R(l, \langle s_A, s_B \rangle, \langle s'_A, s'_B \rangle) :\Leftrightarrow R_A(l, s_A, s'_A) \wedge R_B(l, s_B, s'_B)$.
  - $F := State_A \times F_B$.

Special case where all states of automaton A are accepting.

Synchronized Product of Two Automata

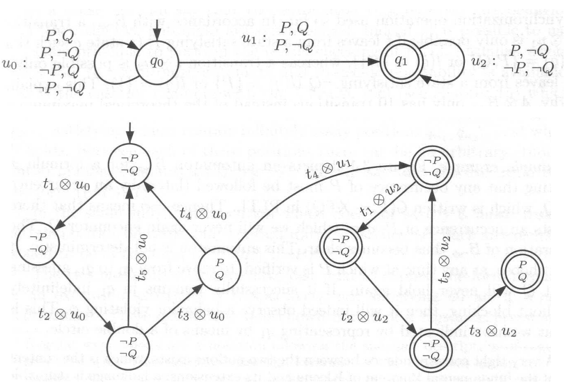


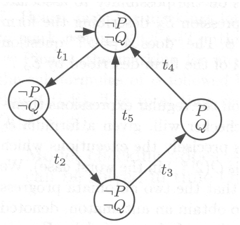
Edmund Clarke: "Model Checking", 1999.



Example

Check whether $S \models \Box(P \Rightarrow \bigcirc\Diamond Q)$.





B. Berard et al: "Systems and Software Verification", 2001.

The product automaton accepts a run, thus the property does not hold.

Checking Emptiness

How to check whether $\mathcal{L}(A)$ is non-empty?

- Suppose $A = \langle I, R, F \rangle$ accepts a run r.
  - Then r contains infinitely many occurrences of some state in F.
  - Since *State* is finite, in some suffix r' every state occurs infinit. often.
  - Thus every state in r' is reachable from every other state in r'.
- C is a strongly connected component (SCC) of graph G if
  - C is a subgraph of G,
  - every node in C is reachable from every other node in C along a path entirely contained in C, and
  - C is maximal (not a subgraph of any other SCC of G).
- Thus the states in r' are contained in an SCC C.
  - C is reachable from an initial state.
  - C contains an accepting state.
  - Conversely, *any* such SCC generates an accepting run.

$\mathcal{L}(A)$ is non-empty if and only if the reachability graph of A has an SCC that contains an accepting state.

Checking Emptiness

Find in the reachability graph an SCC that contains an accepting state.

- We have to find an accepting state with a cycle back to itself.
  - Any such state belongs to some SCC.
  - Any SCC with an accepting state has such a cycle.
  - Thus this is a sufficient and necessary condition.
- Any such a state s defines a counterexample run r.
  - $r = \iota \to \ldots \to s \to \ldots \to s \to \ldots \to s \to \ldots$
  - Finite prefix $\iota \to \ldots \to s$ from initial state ι to s.
  - Infinite repetition of cycle $s \to \ldots \to s$ from s to itself.

This is the core problem of PLTL model checking; it can be solved by a *depth-first search* algorithm.

Basic Structure of Depth-First Search

Visit all states of the reachability graph of an automaton $\langle \{\iota\}, R, F \rangle$.

global
    $\mathit{StateSpace}\ V := \{\}$
    $\mathit{Stack}\ D := \langle\rangle$

proc $\mathit{main}()$
    $\mathit{push}(D, \iota)$
    $\mathit{visit}(\iota)$
    $\mathit{pop}(D)$
end

proc $\mathit{visit}(s)$
    $V := V \cup \{s\}$
    for $\langle l, s, s' \rangle \in R$ **do**
        if $s' \notin V$
            $\mathit{push}(D, s')$
            $\mathit{visit}(s')$
            $\mathit{pop}(D)$
        end
    end
end

State space V holds all states visited so far; stack D holds path from initial state to currently visited state.

Checking State Properties

Apply depth-first search to checking a state property (assertion).

```
global
  StateSpace V := {}
  Stack D := ⟨⟩

proc main()
  // r becomes true, iff
  // counterexample run is found
  push(D, ι)
  r := search(ι)
  pop(D)
end
```

```
function search(s)
  V := V ∪ {s}
  if ¬check(s) then
    print D
    return true
  end
  for ⟨l, s, s'⟩ ∈ R do
    if s' ∉ V
      push(D, s')
      r := search(s')
      pop(D)
      if r then return true end
    end
  end
  return false
end
```

Stack D can be used to print counterexample run.

Depth-First Search for Acceptance Cycle

```
global
  ...
  Stack C := ⟨⟩

proc main()
  push(D, ι); r := search(ι); pop(D)
end

function searchCycle(s)
  for ⟨l, s, s'⟩ ∈ R do
    if has(D, s') then
      print D; print C; print s'
      return true
    else if ¬has(C, s') then
      push(C, s');
      r := searchCycle(s')
      pop(C);
      if r then return true end
    end
  end
  return false
end

boolean search(s)
  V := V ∪ {s}
  for ⟨l, s, s'⟩ ∈ R do
    if s' ∉ V
      push(D, s')
      r := search(s')
      pop(D)
      if r then return true end
    end
  end
  if s ∈ F then
    r := searchCycle(s)
    if r then return true end
  end
  return false
end
```

Depth-First Search for Acceptance Cycle

- At each call of $search(s)$,
  - s is a reachable state,
  - D describes a path from ι to s.
- $search$ calls $searchCycle(s)$
  - on a reachable accepting state s
  - in order to find a cycle from s to itself.
- At each call of $searchCycle(s)$,
  - s is a state reachable from a reachable accepting state s_a,
  - D describes a path from ι to s_a,
  - $D \rightarrow C$ describes a path from ι to s (via s_a).
- Thus we have found an accepting cycle $D \rightarrow C \rightarrow s'$, if
  - there is a transition $s \xrightarrow{l} s'$,
  - such that s' is contained in D.

If the algorithm returns "true", there exists a violating run; the converse follows from the exhaustiveness of the search.

Implementing the Search

- The state space V,
  - is implemented by a hash table for efficiently checking $s' \notin V$.
- Rather than using explicit stacks D and C,
  - each state node has two bits d and c,
  - d is set to denote that the state is in stack D,
  - c is set to denote that the state is in stack C.
- The counterexample is printed,
  - by searching, starting with ι, the unique sequence of reachable nodes where d is set until the accepting node s_a is found, and
  - by searching, starting with a successor of s_a, the unique sequence of reachable nodes where c is set until the cycle is detected.
- Furthermore, it is not necessary to reset the c bits, because
  - *search* first explores all states reachable by an accepting state s before trying to find a cycle from s; from this, one can show that
  - called with the first accepting node s that is reachable from itself, *search2* will not encounter nodes with c bits set in previous searches.
  - With this improvement, every state is only visited twice.

Complexity of the Search

The complexity of checking $S \models P$ is as follows.

- Let $|P|$ denote the number of subformulas of P.
- $|State_{(\neg P)_A}| = O(2^{|P|})$.
- $|State_{A \otimes B}| = |State_A| \cdot |State_B|$.
- $|State_{S_A \otimes (\neg P)_A}| = O(|State_{S_A}| \cdot 2^{|P|})$
- The time complexity of *search* is linear in the size of *State*.
  - Actually, in the number of reachable states (typically much smaller).
  - Only true for the improved variant where the c bits are not reset.
  - Then every state is visited at most twice.

PLTL model checking is linear in the number of reachable states but exponential in the size of the formula.

The Overall Process

Basic PLTL model checking for deciding $S \models P$.

- Convert system S to automaton S_A.
  - Atomic propositions of PLTL formula are evaluated on each state.
- Convert negation of PLTL formula P to automaton $(\neg P)_A$.
  - How to do so, remains to be described.
- Construct synchronized product automaton $S_A \otimes (\neg P)_A$.
  - After that, formula labels are not needed any more.
- Find SCC in reachability-graph of product automaton.
  - A purely graph-theoretical problem that can be efficiently solved.
  - Time complexity is linear in the size of the state space of the system but exponential in the size of the formula to be checked.
  - Weak scheduling fairness with k components: runtime is increased by factor $k+2$ (worst-case, "in practice just factor 2" [Holzmann]).

The basic approach immediately leads to *state space explosion*; further improvements are needed to make it practical.

On the Fly Model Checking

For checking $\mathcal{L}(S_A \otimes (\neg P)_A) = \emptyset$, it is not necessary to construct the states of S_A in advance.

- Only the property automaton $(\neg P)_A$ is constructed in advance.
  - This automaton has comparatively small state space.
- The system automaton S_A is constructed on the fly.
  - Construction is guided by $(\neg P)_A$ while computing $S_A \otimes (\neg P)_A$.
  - Only that part of the reachability graph of S_A is expanded that is consistent with $(\neg P)_A$ (i.e. can lead to a counterexample run).
- Typically only a part of the state space of S_A is investigated.
  - A smaller part, if a counterexample run is detected early.
  - A larger part, if no counterexample run is detected.

Unreachable system states and system states that are not along possible counterexample runs are never constructed.

On the Fly Model Checking

Expansion of state $s = \langle s_0, s_1 \rangle$ of product automaton $S_A \otimes (\neg P)_A$ into the set $R(s)$ of transitions from s (**for** $\langle l, s, s' \rangle \in R(s)$ **do** ...).

- Let S'_1 be the set of all successors of state s_1 of $(\neg P)_A$.
  - Property automaton $(\neg P)_A$ has been precomputed.
- Let S'_0 be the set of all successors of state s_0 of S_A.
  - Computed on the fly by applying system transition relation to s_0.
- $R(s) := \{\langle l, \langle s_0, s_1 \rangle, \langle s'_0, s'_1 \rangle \rangle : s'_0 \in S'_0 \wedge s'_1 \in S'_1 \wedge s_1 \xrightarrow{l} s'_1 \wedge L(s'_0) \in l\}$.
  - Choose candidate $s'_0 \in S'_0$.
  - Determine set of atomic propositions $L(s'_0)$ true in s'_0.
  - If $L(s'_0)$ is not consistent with the label of any transition $\langle s_0, s_1 \rangle \xrightarrow{l} \langle s'_0, s'_1 \rangle$ of the proposition automaton, s'_0 it is ignored.
  - Otherwise, R is extended by every transition $\langle s_0, s_1 \rangle \xrightarrow{l} \langle s'_0, s'_1 \rangle$ where $L(s'_0)$ is consistent with label l of transition $s_1 \xrightarrow{l} s'_1$.

Actually, depth-first search proceeds with first suitable successor $\langle s'_0, s'_1 \rangle$ before expanding the other candidates.

The Model Checker Spin

- Spin system:
  - Gerard J. Holzmann et al, Bell Labs, 1980–.
  - Freely available since 1991.
  - Workshop series since 1995 (12th workshop "Spin 2005").
  - ACM System Software Award in 2001.
- Spin resources:
  - Web site: http://spinroot.com.
  - Survey paper: Holzmann "The Model Checker Spin", 1997.
  - Book: Holzmann "The Spin Model Checker — Primer and Reference Manual", 2004.

Goal: verification of (concurrent/distributed) software models.

The Model Checker Spin

On-the-fly LTL model checking.

- Explicit state representation
  - Representation of system S by automaton S_A.
  - There exist various other approaches (discussed later).
- On-the-fly model checking.
  - Reachable states of S_A are only expended on demand.
  - *Partial order reduction* to keep state space manageable.
- LTL model checking.
  - Property P to be checked described in PLTL.
    - Propositional linear temporal logic.
  - Description converted into property automaton P_A.
    - Automaton accepts only system runs that do not satisfy the property.

Model checking based on automata theory.

The Spin System Architecture

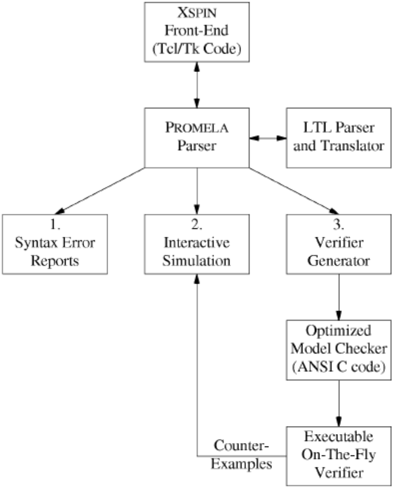


Fig. 1. The structure of SPIN simulation and verification.

Features of Spin

- System description in Promela.
  - Promela = Process Meta-Language.
    - Spin = Simple Promela Interpreter.
  - Express coordination and synchronization aspects of a real system.
  - Actual computation can be e.g. handled by embedded C code.
- Simulation mode.
  - Investigate individual system behaviors.
  - Inspect system state.
  - Graphical interface XSpin for visualization.
- Verification mode.
  - Verify properties shared by all possible system behaviors.
  - Properties specified in PLTL and translated to "never claims".
    - Promela description of automaton for negation of the property.
  - Generated counter examples may be investigated in simulation mode.

Verification and simulation are tightly integrated in Spin.

Some New Promela Features

Active processes, inline definitions, atomic statements, output.

```
mtype = { P, C, N }
mtype turn = P;

inline request(x, y) { atomic { x == y -> x = N } }
inline release(x, y) { atomic { x = y } }
#define FORMAT "Output: %s\n"

active proctype producer()
{
  do
  :: request(turn, P) -> printf(FORMAT, "P"); release(turn, C);
  od
}

active proctype producer()
{
  do
  :: request(turn, C) -> printf(FORMAT, "C"); release(turn, P);
  od
}
```

Some New Promela Features

Embedded C code.

```
/* declaration is added locally to proctype main */
c_state "float f" "Local main"

active proctype main()
{
  c_code { Pmain->f = 0; }
  do
    :: c_expr { Pmain->f <= 300 };
       c_code { Pmain->f = 1.5 * Pmain->f ; };
       c_code { printf("%4.0f\n", Pmain->f); };
  od;
 }
```

Can embed computational aspects into a Promela model (only works in verification mode where a C program is generated from the model).

Spin Usage for Simulation

Command-line usage of spin: `spin --`.

- Perform syntax check.

  `spin -a` *file*

- Run simulation.

No output:	`spin` *file*
One line per step:	`spin -p` *file*
One line per message:	`spin -c` *file*
Bounded simulation:	`spin -u`*steps* *file*
Reproducible simulation:	`spin -n`*seed* *file*
Interactive simulation:	`spin -i` *file*
Guided simulation:	`spin -t` *file*

Spin Usage for Verification

- Generate never claim

  spin -f "*nformula*" >*neverfile*

- Generate verifier.

  spin -N *neverfile* -a *file*

```
ls -la pan.*
-rw-r--r--  1 schreine schreine   3073 2005-05-10 16:36 pan.b
-rw-r--r--  1 schreine schreine 150665 2005-05-10 16:36 pan.c
-rw-r--r--  1 schreine schreine   8735 2005-05-10 16:36 pan.h
-rw-r--r--  1 schreine schreine  14163 2005-05-10 16:36 pan.m
-rw-r--r--  1 schreine schreine  19376 2005-05-10 16:36 pan.t
```

- Compile verifier.

  cc -O3 -DNP -DMEMLIM=128 -o pan pan.c

- Execute verifier.

Options:	./pan --
Find non-progress cycle:	./pan -l
Weak scheduling fairness:	./pan -l -f
Maximum search depth:	./pan -l -f -m*depth*

Spin Verifier Generation Options

`cc -O3` *`options`* `-o pan pan.c`

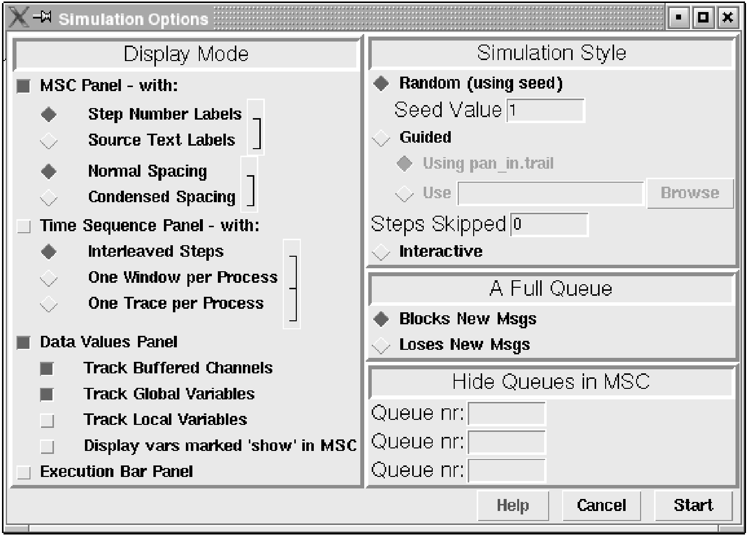
`-DNP`	Include code for non-progress cycle detection
`-DMEMLIM=`*`N`*	Maximum number of MB used
`-DNOREDUCE`	Disable partial order reduction
`-DCOLLAPSE`	Use collapse compression method
`-DHC`	Use hash-compact method
`-DDBITSTATE`	Use bitstate hashing method

For detailed information, look up the manual.

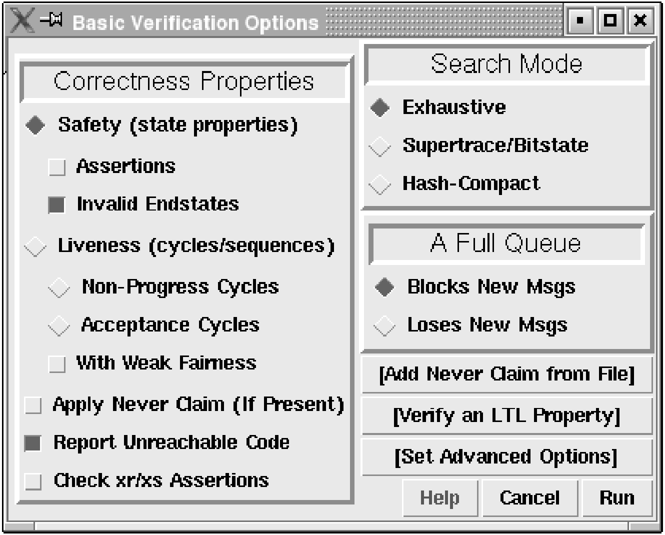
Spin Verifier Output

```
warning: for p.o. reduction to be valid the never claim must be stutter-invariant
(never claims generated from LTL formulae are stutter-invariant)
(Spin Version 4.2.2 -- 12 December 2004)
        + Partial Order Reduction

Full statespace search for:
        never claim             +
        assertion violations    + (if within scope of claim)
        acceptance   cycles     + (fairness disabled)
        invalid end states      - (disabled by never claim)

State-vector 52 byte, depth reached 587, errors: 0
     861 states, stored
     856 states, matched
    1717 transitions (= stored+matched)
       0 atomic steps
hash conflicts: 1 (resolved)

Stats on memory usage (in Megabytes):
...
2.622   total actual memory usage
...
```

XSpin Simulation Options



XSpin Verification Options



Other Approaches to Model Checking

There are fundamentally different approaches to model checking than the automata-based one implemented in Spin.

- Symbolic Model Checking (e.g. SMV, NuSMV).
  - Core: binary decision diagrams (BDDs).
    - Data structures to represent boolean functions.
    - Can be used to describe state sets and transition relations.
  - The set of states satisfying a CTL formula P is computed as the BDD representation of a fixpoint of a function (predicate transformer) F_P.
    - If all initial system states are in this set, P is a system property.
  - BDD packages for efficiently performing the required operations.
- Bounded Model Checking (e.g. NuSMV2).
  - Core: propositional satisfiability.
    - Is there a truth assignment that makes propositional formula true?
  - There is a counterexample of length at most k to a LTL formula P, if and only if a particular propositional formula $F_{k,P}$ is satisfiable.
    - Problem: find suitable bound k that makes method complete.
  - SAT solvers for efficiently deciding propositional satisfiability.

Other Approaches to Model Checking

- Counter-Example Guided Abstraction Refinement (e.g. BLAST).
  - Core: model abstraction.
    - A finite set of predicates is chosen and an abstract model of the system is constructed as a finite automaton whose states represent truth assignments of the chosen predicates.
  - The abstract model is checked for the desired property.
    - If the abstract model is error-free, the system is correct; otherwise an abstract counterexample is produced.
    - It is checked whether the abstract counterexample corresponds to a real counterexample; if yes, the system is not correct.
    - If not, the chosen set of predicates contains too little information to verify or falsify the program; new predicates are added to the set. Then the process is repeated.
  - Core problem: how to refine the abstraction.
    - Automated theorem provers are applied here.

Many model checkers for software verification use this approach.