A Process Calculus II

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Equational Laws

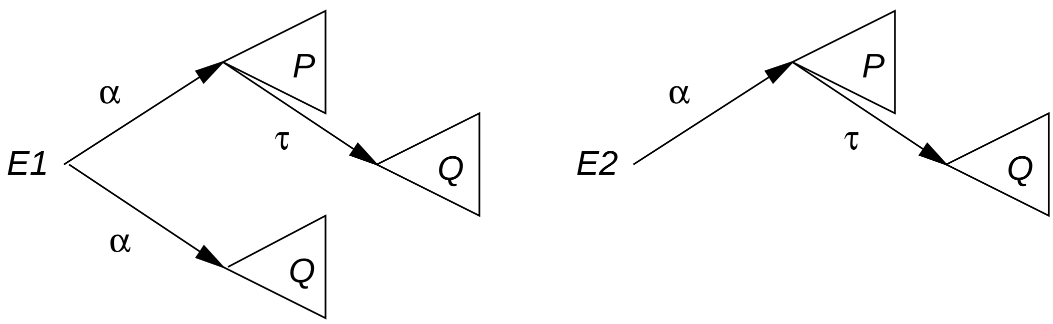
- Static laws
  - Static combinators: composition, restriction, labelling.
  - Action rules do not change graph structure.
  - Algebra of flow graphs.
- Dynamic laws
  - Dynamic combinators: prefix, summation, constants.
  - Action rules change graph structure.
  - Algebra of transition graphs.
- Expansion law
  - Relating static laws to dynamic laws.

Static Laws

- Composition laws
  - $P|Q = Q|P$
  - $P + (Q|R) = (P|Q)|R$
  - $P|0 = P$
- Restriction laws
  - $P\backslash L = P$, if $L(P) \cap (L \cup \overline{L}) = \{\ \}$.
  - $P\backslash K\backslash L = P\backslash(K \cup L)$
  - ...
- Relabelling laws
  - $P[\mathit{Id}] = P$
  - $P[f][f'] = P[f' \circ f]$
  - ...

Dynamic Laws

- Monoid laws
  - $P + Q = Q + P$
  - $P + (Q + R) = (P + Q) + R$
  - $P + P = P$
  - $P + 0 = P$
- τ laws
  - $\alpha.\tau.P = \alpha.P$
  - $P + \tau.P = \tau.P$
  - $\alpha.(P + \tau.Q) + \alpha.Q = \alpha.(P + \tau.Q)$



- Transition Relation $P \stackrel{\alpha}{\Rightarrow} P'$
  - $P \stackrel{\alpha}{\Rightarrow} P' \equiv P(\stackrel{\tau}{\rightarrow})^* \stackrel{\alpha}{\rightarrow} (\stackrel{\tau}{\rightarrow})^*\ P'$

Non-Laws

- $\tau.P = P$
  - A $= a.A + \tau.b.A$
  - A' $= a.A' + b.A'$
  - A may switch to state in which only b is possible.
  - B *always* allows a or b.
  - Action sequence a, a may yield dedlock for right side.
- $\alpha.(P + Q) = \alpha.P + \alpha.Q$
  - $a.(b.P + c.Q) = a.b.P + a.c.Q$
  - $b.P$ is a-derivative of right side, not capable of c action.
  - a-derivative of left side is capable of c action!
  - Action sequence a, c may yield deadlock for right side.

The Expansion Law

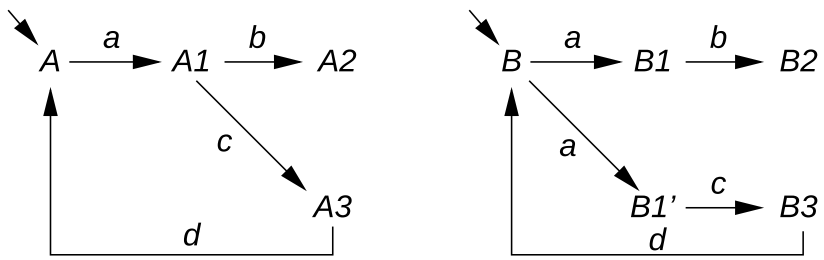
- The Expansion Law
  - Let $P \equiv (P_1[f_1]| \dots |P_n[f_n])\backslash L$
  - $P = \Sigma\{f_1(\alpha).(P_1[f_1]| \dots |P_i'[f_i]| \dots |P_n[f_n])\backslash L$:
    $P_i \stackrel{\alpha}{\to} P_i'$, $f_i(\alpha)$ not in $L \cup \overline{L}\}$
    $+ \Sigma\{\tau.(P_1[f_1]| \dots |P_i'[f_i]| \dots |P_j'[f_j]| \dots |P_n[f_n])\backslash L$:
    $P_i \stackrel{l_1}{\to} P_i'$, $P_j \stackrel{l_2}{\to} P_j'$, $f_i(l_1) = \overline{f_i(l_2)}$, $i < j\}$
- Corollary
  - Let $P \equiv (P_1| \dots |P_n)\backslash L$
  - $P = \Sigma\{\alpha.(P_1| \dots |P_i'| \dots |P_n)\backslash L$:
    $P_i \stackrel{\alpha}{\to} P_i'$, αnot in $L \cup L'\}$
    $+ \Sigma\{\tau.(P_1| \dots |P_i'| \dots |P_j'| \dots P_n)\backslash L$:
    $P_i \stackrel{l}{\to} P_i'$, $P_j \stackrel{\overline{l}}{\to} P_j'$, $i < j\}$
- Example
  - $P_1 = a.P_1' + b.P_1''$
  - $P_2 = \overline{a}.P_2' + c.P_2''$
  - $(P_1|P_2)\backslash a = b.(P_1''|P_2)\backslash a + c.(P_1|P_2'')\backslash a + \tau.(P_1'|P_2')\backslash a$

Equivalence of Agents

- Equivalence
  - Two agents P and Q are different if distinction can be detected by external agent interacting with them
- *Strong* equivalence:
  - τ is treated like any other (observable) action.
- *Observation* equivalence:
  - τ cannot be observed by external agent.
  - *Congruence relation* i.e. preserved by all algebraic contexts.

Experimenting upon Agents

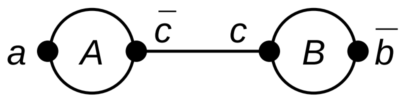
- Example agents A and B
  - $A = a.(b.0 + c.d.A)$
  - $B = a.b.0 + a.c.d.B$

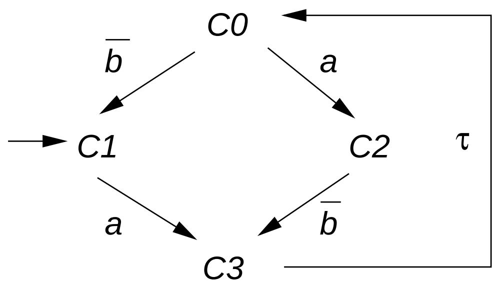


- "Language understood" by A and B
  - $(a.c.d)^*.a.b.0$
  - A and B seem equivalent.
- Ports a, b, c, d.
  - Initially only a is "unlocked".
  - Observer "presses button" a.
  - In A, b and c are "unlocked".
  - In B, sometimes b, sometimes c is "unlocked".
  - A and B can be experimentally distinguished!

Strong Bisimulation

- *Strong bisimulation*
  - Binary relation S over agents such that $(P, Q) \in S$ implies
  - If $P \overset{\alpha}{\to} P'$, then $Q \overset{\alpha}{\to} Q'$ with $(P', Q') \in S$ and vice versa.
  - For every action α, every α-derivative of P is equivalent to some α-derivative of Q.
- Example





  - Claim: $(A|B)\backslash c = C_1$
  - True if S is a strong bisimulation:
    $S = \{\ ((A|B)\backslash c,\ C_1),\ ((A'|B)\backslash c,\ C_3),$
    $((A|B')\backslash c,\ C_0),\ ((A'|B')\backslash c,\ C_2)\ \}$
  - Check derivatives of each of the eight agents.

Strong Equivalence

- *Strong equivalence* $P{\sim}Q$
  - $P{\sim}Q$, if $(P,Q) \in S$ for some strong bisimulation S.
  - $\sim\, = \cup\{S\colon S$ is a strong bisimulation$\}$.
- Corollaries:
  - $\sim$ is the largest strong bisimulation.
  - $\sim$ is an equivalence relation.
- Proposition:
  - $P{\sim}Q$ iff, for all α,
  - If $P \stackrel{\alpha}{\rightarrow} P'$, then $Q \stackrel{\alpha}{\rightarrow} Q'$ with $(P',Q') \in S$ and vice versa.

Properties of Strong Equivalence

- Monoid laws:
  - $P + Q \sim Q + P$
  - $P + (Q + R) \sim (P + Q) + R$
  - $P + P \sim P$
  - $P + 0 \sim P$
- Static laws:
  - $P|Q \sim Q|P$
  - $P + (Q|R) \sim (P|Q)|R$
  - $P|0 \sim P$
  - $\ldots$
- The Expansion law:
  - $\ldots$ ($=$ replaced by $\sim$)

Strong Congruence

- Strong congruence
  - Strong equivalence is substitutive under all combinators and under recursive definitions
- Let $P_1 \sim P_2$
  - $\alpha.P_1 \sim \alpha.P_2$
  - $P_1 + Q \sim P_2 + Q$
  - $P_1|Q \sim P_2|Q$
  - $P_1 \backslash L \sim P_2 \backslash L$
  - $P_1[f] \sim P_2[f]$

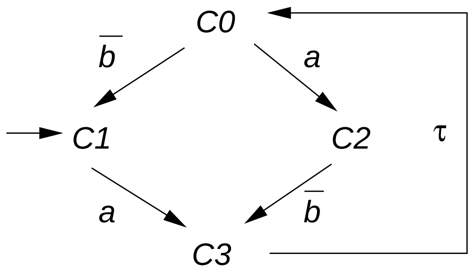
Bisimulation and Observation Equivalence

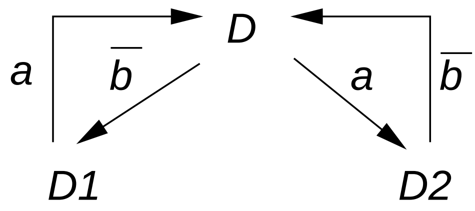
- (Weak) bisimulation and (observation) equivalence:
  - τ action may be matched by zero or more τ actions.
- Auxiliary definitions:
  - $\hat{t}$ is the action sequence gained by deleting all occurences of τ from t.
  - $E \xrightarrow{t} E'$, if $t = \alpha_1 \ldots \alpha_n$ and $E \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} E'$.
  - $E \overset{t}{\Rightarrow} E'$ if $t = \alpha_1 \ldots \alpha_n$ and $E (\xrightarrow{\tau})^* \xrightarrow{\alpha_1} (\xrightarrow{\tau})^* \ldots (\xrightarrow{\tau})^* \xrightarrow{\alpha_n} (\xrightarrow{\tau})^* E'$.
  - E' is a t-descendant of E iff $E \overset{\hat{t}}{\Rightarrow} E'$.
- Relationship
  - $P \xrightarrow{t} P'$ implies $P \overset{t}{\Rightarrow} P'$ implies $P \xrightarrow{\hat{t}} P'$

Weak Bisimulation and Observation Equivalence

- *(Weak) bisimulation*
  - Binary relation S such that $(P, Q) \in S$ implies
  - if $P \xrightarrow{\alpha} P'$, then $Q \stackrel{\widehat{\alpha}}{\Rightarrow} Q'$ with $(P', Q') \in S$ (and vice versa).
- *Observation equivalence* $P{\approx}Q$
  - $P{\approx}Q$ if $(P, Q) \in S$ for some weak bisimulation S.
  - $\approx\ = \cup\{S : S$ is a weak bisimulation$\}$

Examples





- Agents C_0 and D
  - Bisimulation $S = \{(C_0, D), (C_1, D_1), (C_2, D_2), (C_3, D)\}$
  - No *strong* bisimulation containing (C_3, D) since $C_3 \xrightarrow{\tau} C_0$ but there is no $D \xrightarrow{\tau} D'$.
- Agents A and B
  - $A_0 = a.A_0 + b.A_1 + \tau.A_1$
    $A_1 = a.A_1 + \tau.A_2$
    $A_2 = b.A_0$
  - $B_1 = a.B_1 + \tau.B_2$
    $B_2 = b.B_1$
  - Bisimulation $S = \{ (A_0, B_1), (A_1, B_1), (A_2, B_2) \}$ (note that $B_1 \overset{b}{\Rightarrow} B_1$!)

Properties of Bisimulation

- Propositions:
  - $\approx$ is the largest bisimulation.
  - $\approx$ is an equivalence relation.
  - $P \approx \tau.P$
- $\approx$ is *not* yet equality:
  - $\approx$ not preserved by summation.
  - $a.0 + b.0 \approx a.0{+}\tau.b.0$ does *not* hold!
  - Proof: if (P,Q) were in a bisimulation S, then, since $Q \stackrel{\tau}{\rightarrow} b.0$, we need $(P', b.0)$ in S with $P \stackrel{\epsilon}{\Rightarrow} Q'$. But the only P' is P itself but $(P, b.0)$ can be not in S, since $P \stackrel{a}{\rightarrow} 0$, while $b.0$ has no a-descendant.
- Relations:
  - $P{\sim}Q$ implies $P{=}Q$ implies $P{\approx}Q$

Equality not yet fully captured.

Observation Congruence

- *Stability*:
  - P is stable if P has no τ-derivative.
- Derivation of equality:
  - If $P \approx Q$ and both are stable, then $P = Q$.
  - If $P \approx Q$ then $\alpha.P = \alpha.Q$
- $P = Q$ (*observation congruence*)
  - If $P \overset{\alpha}{\to} P'$, then $Q \overset{\alpha}{\Rightarrow} Q'$ with $P' \approx Q'$ (and vice versa).
  - Preserved under all process operators.

Observation congruence is the equality of the process algebra.

Summary

- Algebraic approach to semantics of parallel programs.
  - Processes are algebraic terms.
  - Calculus for term manipulation preserving equality.
  - Main interest: how do processes interact with each other?
- Central notions:
  - Strong bisimilarity: equivalence even for internal actions.
  - Observation equivalence: equivalence only for observable actions.
  - Observation congruence: equivalence preserved under all substitutions.

Modeling of systems that react with their environment.