Formal Foundations of Computer Science 1

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Course Organization

- Lectures
  1. Option: Tuesday, 8:15–9:45, HS 12.
  2. Option: Tuesday, 18:30–20:00, HS 5.
- Information
  - Web: `http://www.risc.uni-linz.ac.at/courses/formal`.
  - Office: Tuesday, 17:00–18:00, KG, 4th floor, RISC-Linz rooms.
- Grades
  - Written exam at end of semester.

Presence is strongly recommended.

Exercises

- Begin: October 13 and 15.
  - Ordered hardcopies of lecture notes.
  - Master hardcopy of first exercise sheet.
- Late Registration
  - email: Wolfgang.Schreiner@risc.uni-linz.ac.at
  - Firstname, Lastname, Mat.Nr., SKZ, time constraints.
- Grades
  - 10 exercise sheets.

Presence is mandatory.

Course Materials

- Lecture notes.
  - Web page (HTML, PostScript, PDF).
  - Hardcopy (order now: ÖS 50,-).
  - Explains details for home study.
- Course slides.
  - Web page (HTML, PostScript, PDF).
  - Print yourself and bring to lecture.
  - Explains basics; use for annotations.

Listen during lecture rather than write!

Logic Evaluator

Interpreter for executing mathematical definitions.

```
pred <(m, n) <=> and(<=(m, n), not(=(m, n)));
> predicate </2.
pred divides(n, m) <=> exists(p in nat(1, m): =(*(n, p), m));
> predicate divides/2.
pred isPrime(p) <=>
  and(<(1, p),
      forall(n in nat(2, -(p, 1)): not(divides(n, p))));
> predicate isPrime/1.
pred isNextPrime(n, p) <=>
  and(<=(n, p), isPrime(p),
      not(exists(q in nat(n, -(p, 1)): isPrime(q))));
> predicate isNextPrime/2.
formula isPrime(19);
> true.
formula isNextPrime(14, 19);
> false.
```

```
formula isNextPrime(14, 17);
```

Contents

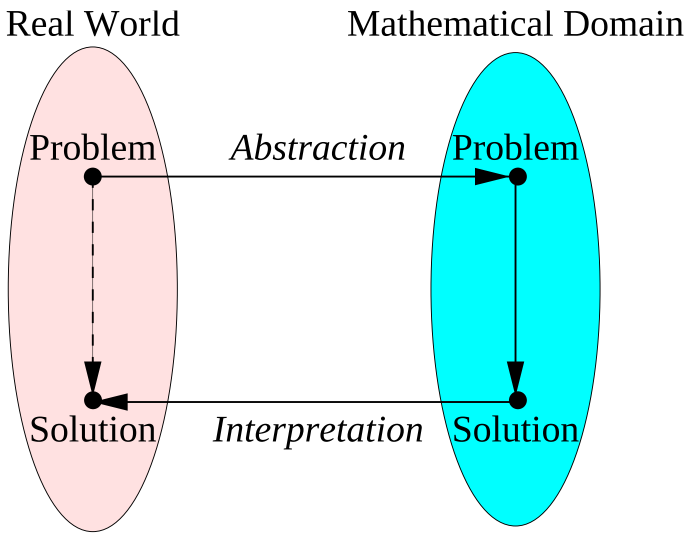
Introduction into basic mathematical domains and techniques as a foundation for computer science.

1. The Language of Logic,
2. Sets, Functions, and Relations,
3. Numbers,
4. Induction and Recursion.
5. More on Functions,
6. More on Relations.

A Defining New Notions,

B Proving Propositions.

Mathematical Domains



- **Abstraction**: construct a simpler image of the problem.

  Simpler but precisely defined; operation in well understood domains possible.

- **Interpretation**: translate solution back into real world.

  Effects as predicted by knowledge about mathematical domain.

Mathematical Techniques

- Precise language.
  - Exact definition of objects.
  - Exact statements of facts.
- Correct reasoning.
  - Rules for thinking.
  - Rules for arguing.

These techniques enable us to understand definitions and to judge the correctness of arguments in any scientific domain.

Our Goals

- Insight into mathematical domains.
  - Construction.
  - Properties.
  - Operation.
- Mastering of mathematical techniques.
  - Defining.
  - Proving.

In-depth understanding, not just learning by heart.