1. Exercises

(1) Prove that if G is a group, X is a set, $\varphi : G \times X \to X$ a group action then
$$y \in Orbit(\varphi, x) \Leftrightarrow Orbit(\varphi, x) = Orbit(\varphi, y)$$
(2) For G, X, φ as in the previous exercise prove that G_x is a subgroup of G.
(3) For G a group, H a subgroup of G prove that $g_1H = g_2H$ or $g_1H \cap g_2H = \emptyset$.
(4) For G, H as above with G finite. Prove that $|g_1H| = |g_2H|$ for all $g_1, g_2 \in G$.