1. How many subsets of the set $[10] = \{1, 2, \ldots, 10\}$ contain at least one odd integer?

2. Let $m, n \in \mathbb{N}$. Give a combinatorial proof of the identity

$$\left(\!\!\binom{n}{k}\!\!\right) = \left(\!\!\binom{k+1}{n-1}\!\!\right).$$

3. Show that

$$k!\, S_2(n,k) = \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n, \qquad n, k \geq 0.$$

   Hint: show that both sides satisfy the same recurrence.

4. Prove the Theorem 15 by induction, i.e., show that

$$x^n = \sum_{k=0}^{n} S_2(n,k) x^{\underline{k}}, \qquad n \geq 0.$$