37. Let $T_n$ be the number of tilings of a $3 \times n$ rectangle with straight trominoes (i.e., $1 \times 3$ and $3 \times 1$ pieces).

    (a) Determine a recurrence relation for $T_n$.

    (b) Express the partial sum $s_n = \sum_{k=0}^{n} T_k$ in terms of $T_n$.

38. Express $s_n = \sum_{k=0}^{n} a_k$ in terms of $a_n, a_{n+1}, \ldots$, where the sequence $\big(a_n\big)_{n\geq 0}$ is given by the recurrence

$$a_{n+3} = 5a_{n+1} - 4a_n, \qquad a_0 = a_1 = 1, \quad a_2 = 2.$$

39. Compute a hypergeometric closed form of the sum $s_n = \sum_{k=0}^{n} \frac{4}{(2k-1)(2k+1)}$ by applying Gosper's algorithm.

40. Show that the harmonic numbers $(H_n)_{n\geq 0}$ are not hypergeometric using Gosper's algorithm.

    *Hint:* You can use the fact that if Gosper's algorithm does not find a telescoper, then the sum cannot be expressed as a linear combination of a fixed number of hypergeometric sequences.