29. Suppose the holonomic function $f(x) = \sum_{n\geq 0} a_n x^n$ satisfies the linear differential equation
$$xf(x) + (x+1)f'(x) - f''(x) = 0.$$
Derive a linear recurrence with polynomial coefficients for the coefficient sequence $(a_n)_{n\geq 0}$. Check whether your result agrees with the bounds on the order and degree given in Theorem 57.

30. The Catalan numbers C_n satisfy the linear recurrence
$$-(4n+2)C_n + (n+2)C_{n+1} = 0.$$
Derive a linear differential equation with polynomial coefficients for the generating function $f(x) = \sum_{n\geq 0} C_n x^n$.

31. Let $f(x) = \sin(x), g(x) = e^{-x^2+x+1}$ be the holonomic functions satisfying the differential equations
$$f(x) + f''(x) = 0, \quad (1-2x)g(x) - g'(x) = 0.$$
Find a linear differential equation with polynomial coefficients for the function $h(x) = f(x) + g(x)$.

Hint: The same idea as in Example 44 can be used. Instead of considering shifts, you can consider derivatives and instead of solving a linear system over $\mathbb{Q}$, a linear system over $\mathbb{Q}(x)$ has to be solved.

32. Find a linear differential equation with polynomial coefficients for the holonomic functions $f'(x)$ and $\int_x f(x)$ where f is the function from Exercise 29, i.e., it satisfies
$$xf(x) + (x+1)f'(x) - f''(x) = 0.$$