17. Let x be an indeterminate and $n \in \mathbb{N}$. Show that

    (a) $x^n = \sum_{k=0}^{n} S_2(n,k) x^{\underline{k}}$,

    (b) $x^{\underline{n}} = \sum_{k=0}^{n} S_1(n,k) x^k$.

18. Let the sequence $\left(f_n\right)_{n\geq 0}$ be recursively defined by

$$f_{n+3} - 2f_{n+2} - 5f_{n+1} + 6f_n = 0, \qquad f_0 = 0,\ f_1 = 8,\ f_2 = 2.$$

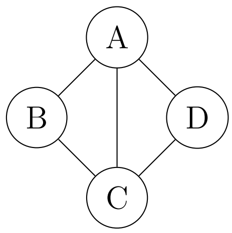
    What is the companion matrix of this recurrence? Determine the solution of the recurrence analogously to the Fibonacci example presented in the lecture.

19. Let a_n denote the number of distinct paths of length $n+1$ from vertex A to vertex B in the graph displayed below.

    (a) Determine a recurrence for a_n.

    (b) Determine a closed form expression of a_n.

    (c) Determine the generating function of a_n.



20. Prove Theorem 40 for recurrences of order two, i.e., for $c_0 \neq 0, c_1 \in \mathbb{K}$ with

$$x^2 + c_1 x + c_0 = (x - \alpha_1)(x - \alpha_2)$$

    with $\alpha_1 \neq \alpha_2 \in \mathbb{K}$, show that $(\alpha_1^n)_{n\geq 0}, (\alpha_2^n)_{n\geq 0}$ form a basis for the solutions of the recurrence

$$a_{n+2} + c_1 a_{n+1} + c_0 a_n = 0, \qquad n \geq 0,$$

    and that if $\alpha_1 = \alpha_2 = \alpha$ then $(\alpha^n)_{n\geq 0}, (n\alpha^n)_{n\geq 0}$ form a basis for the solutions of the above recurrence.