

5. Determine the generating function $F(x) = \sum_{n \geq 0} c_n x^n$ for
- (a) $c_n = n^2$.
 - (b) $c_n = \frac{1}{n+1}$.
6. Show that $(\mathbb{K}^{\mathbb{N}}, +, \cdot)$ is a commutative ring with one.
7. Show that $(\mathbb{K}[[x]], +, \cdot)$ is an integral domain.
8. Show that the map $D_x : \mathbb{K}[[x]] \rightarrow \mathbb{K}[[x]]$ defined as

$$D_x \sum_{n=0}^{\infty} a_n x^n := \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n,$$

turns $\mathbb{K}[[x]]$ into a differential ring.