

1. Let $f : \mathbb{Z} \rightarrow \mathbb{C}$ and $a, b \in \mathbb{Z}$ with $a \leq b$.

(a) Show that

$$\sum_{k=a}^b (f(k+1) - f(k)) = f(b+1) - f(a).$$

(b) Assume additionally that $f(k) \neq 0$ for all $a \leq k \leq b$. Show that

$$\prod_{k=a}^b \frac{f(k+1)}{f(k)} = \frac{f(b+1)}{f(a)}.$$

2. Determine a closed form representation for the following expressions:

(a) $\sum_{k=1}^n k^2$

(b) $\sum_{k=1}^n k^3$

(c) $p(n) = \prod_{k=2}^n \left(1 - \frac{1}{k^2}\right)$

3. Describe a worst-case scenario for Quicksort. What is the number of comparisons needed in that case?

4. Let

$$g(n) = \sum_{k=0}^{n-1} \frac{2k}{(k+1)(k+2)}.$$

Show that

$$g(n) = 2H_n + \frac{4}{n+1} - 4.$$