43. Compute a hypergeometric closed form of the sum $s_n = \sum_{k=0}^{n} \frac{4}{(2k-1)(2k+1)}$ by applying Gosper's algorithm.

44. Show that the harmonic numbers $(H_n)_{n\geq 0}$ are not hypergeometric using Gosper's algorithm.

45. Use Zeilberger's algorithm as presented in the lecture to determine a recurrence satisfied by $\sum_{k=0}^{n} \binom{n}{k} k$. You may use the information that the recurrence is of order one with linear coefficients.

46. Given the following holonomic sequence

$$(4n+1)g_{n+2} + 2(4n-1)g_{n+1} - 3(4n+5)g_n = 0, \quad g_0 = 1,\ g_1 = 0.$$

Use Petkovšek's algorithm to compute a closed form of g_n.

47. Petkovšek's algorithm finds the solutions 3^n and $n!$ to the recurrence

$$(n-2)a_{n+2} - (n^2+3n-7)a_{n+1} + 3(n^2-1)a_n = 0.$$

Compute the two factorizations of the operator corresponding to this recurrence.

48. Which algorithms presented in the lecture can be used to solve the following sums? What does "solve" mean for each sequence?

(a) $\sum_{k=-7}^{n} (P_{2k+1} + F_{2k})^2$ where F_n denotes the Fibonacci numbers and P_n the Perrin numbers,

(b) $\sum_{k=0}^{n+7} \frac{\binom{2k}{k}^2}{(k+1)4^k}$,

(c) $\sum_{k=0}^{\infty} (-1)^k \binom{n+1}{k} \binom{2n-2k+1}{n}$,

(d) $\sum_{k=0}^{2n+1} (-2k^2 + 17k + 3)$,

(e) $\sum_{k=0}^{n} \frac{1}{n^2+k^2+1}$,

(f) $\sum_{k=0}^{n} \frac{1}{k^2+\sqrt{5}k-1}$,

(g) $\sum_{k=0}^{2n} (-1)^k \binom{2n}{k}^3$.