

33. Determine all polynomial solutions of the recurrence

$$(4n + 9)a(n) - 4(n + 1)a(n + 1) + 3a(n + 2) = 0.$$

34. Use the results from Chapter 6 to prove that the sequence of harmonic numbers $(H_n)_{n \geq 0}$ is not a polynomial sequence.

35. Compute $\sum_{k=0}^n (2k^3 - 3k^2 + 1)$ both using

(a) falling factorial representation and

(b) interpolation.

36. Implement a program that sums a given polynomial sequence using

(a) falling factorial representation and

(b) interpolation (you may use built-in commands to execute the interpolation, e.g., the command `InterpolatingPolynomial` in Mathematica or the command `lagrange_polynomial` in Sage).

Compute some test cases, in particular compare the timings for the sparse polynomial $729x^{123} - 1$ and the dense polynomial given in `testcase.txt`.