9. Prove Theorem 18: Show that for the formal derivation and integration as defined in the lecture and for all $a(x) \in \mathbb{K}[\![x]\!]$ it holds that

   (1) $D_x \int_x a(x) = a(x)$

   (2) $\int_x D_x a(x) = a(x) - a(0)$

   (3) $[x^n]a(x) = \frac{1}{n!} \left(D_x^n a(x)\right) \big|_{x=0}$

10. Two sequences $(a_n)_{n\geq 0}, (b_n)_{n\geq 0}$ which are non-zero from some index n_0 on are called asymptotically equivalent if $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$. We write $a_n \sim b_n (n \to \infty)$.

    (1) Show that $\sim$ is an equivalence relation.

    (2) Show that for every rational function $r(x) \in \mathbb{K}(x)$ there is a $c \in \mathbb{K}$ and $k \in \mathbb{Z}$ such that $r(n) \sim cn^k$.

    (3) Show that the harmonic numbers H_n cannot be expressed as a rational function, i.e., there is no rational function $r(x) \in \mathbb{K}(x)$ such that $r(n) = H_n$ for all $n \in \mathbb{N}$.

11. Prove Theorem 19 (Multiplicative Inverse):
    Let $a(x) \in \mathbb{K}[\![x]\!]$. Then, there exists a series $b(x) \in \mathbb{K}[\![x]\!]$ with $a(x)b(x) = 1$ if and only if $a(0) \neq 0$.

12. Let $\left(a_n(x)\right)_{n\geq 0}, \left(b_n(x)\right)_{n\geq 0}$ be convergent sequences of formal power series with respective limits $a(x), b(x) \in \mathbb{K}[\![x]\!]$.

    Show that then also $\left(c_n(x)\right)_{n\geq 0}$ with $c_n(x) = a_n(x) + b_n(x)$ is a convergent sequence of formal power series with limit $a(x) + b(x)$.