49. Which of the following power series in $\mathbb{Q}[\![x]\!]$ are holonomic?

    (a) $\exp\left(\frac{x}{\sqrt{1-4x}}\right)$

    (b) $\sum_{n\geq 0}\left(\sum_{k=0}^{n} F_k\binom{n}{k}\right)x^n$, where F_n denotes the Fibonacci numbers.

50. Use $[x^n]\frac{x^k}{(1-x)^{k+1}} = \binom{n}{k}$ to show that for all $a \in \mathbb{K}[\![x]\!]$,

$$[x^n]\frac{1}{1-x}a\left(\frac{x}{x-1}\right) = \sum_{k=0}^{n}(-1)^k\binom{n}{k}[x^k]a(x), \qquad n \geq 0.$$

    This is known as the *Euler transform.*

51. Let $a \in \mathbb{K}[\![x]\!]$ be algebraic and let $b \in \mathbb{K}[\![x]\!]$ with $b(0) = 0$ be s.t. $a(b(x)) = x$. Show that $b(x)$ is algebraic.

52. Let

$$f(x) = (1-x)^{5/2} + \frac{2}{\sqrt{1-x}}\log(1-x)^2 = \sum_{n\geq 0} a_n x^n.$$

    Determine the asymptotic behaviour of a_n.