45. Given is the following P-finite recurrence

$$(4n+1)g_{n+2} + 2(4n-1)g_{n+1} - 3(4n+5)g_n = 0, \quad g_0 = 1,\ g_1 = 0.$$

Determine all candidate pairs for $u(n), v(n+2)$ in Petkovšek's algorithm presented in the lecture and at least one hypergeometric solution to this recurrence.

46. Hyper finds the solutions 3^n and $n!$ to the recurrence

$$(n-2)a(n+2) - (n^2+3n-7)a(n+1) + 3(n^2-1)a(n) = 0.$$

Compute the two factorizations of the operator corresponding to this recurrence.

47. Harmonic numbers are a solution to the recurrence

$$(n+2)a_{n+2} - (2n+3)a_{n+1} + (n+1)a_n = 0,$$

see also Example 52 in the lecture notes. Hyper returns the shift quotient $\{1\}$ as the only solution to this recurrence. Use this to show that Harmonic numbers are not hypergeometric.

48. Chebyshev polynomials of the first kind can be defined using the recurrence

$$T_{n+2}(x) - 2xT_{n+1}(x) + T_n(x) = 0, \quad T_0(x) = 1,\ T_1(x) = x.$$

Compute the generating function $F(z) = \sum_{n\geq 0} T_n(x)z^n$.